JNU MCA
Solved Paper 2007

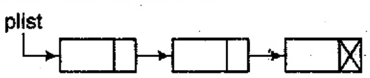
1. Consider the following functions of a complex variable $f_1(z) = \begin{cases} (R_e(z^2))^2/|z|^2, & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$ and $f_2(z) = |z|^2$,
where $R_e(z)$ is the real part of z. Let the two statements
I. $f_1(z)$ is continuous at $z = 0$
II. $f_2(z)$ is analytic at $z = 0$
Which of the following, identify the correct statement?
(a) I is true but II is false
(b) II is true but I is false
(c) Both I and II are true
(d) Both I and II are false

2. If X is uniformly distributed over $(0, 10)$, the probability that $1 < X < 6$ is
(a) 3/10 (b) 1/10
(c) 5/10 (d) None of these

3. The area of triangle formed by the vertices $(p, q + r)$, $(q, p + r)$, $(r, p + q)$ is
(a) $p + q + r$ (b) $pq + qr + rp$
(c) 0 (d) None of these

4. The volume of the tetrahedron included between the plane $3x + 4y - 5z = 60$ and the coordinate planes in cubic units is
(a) 60 (b) 600
(c) 720 (d) None of these

5. What would happen if we apply the following statements to the linked list as below?



temp = plist
loop (temp-> link not null)
temp = temp-> link
endloop
temp-> link = plist

(a) List will become circular
(b) Last element will be deleted
(c) plist will point to last node
(d) None of the above

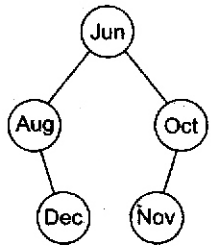
6. The length of the line joining two points on the parabola $y^2 = x$ which is bisected at (1, 2), is
(a) $\sqrt{51}$ (b) $3\sqrt{51}$
(c) $4\sqrt{51}$ (d) None of these

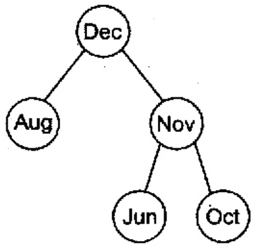
7. If $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{a} \cdot \mathbf{b} = 1$, $\mathbf{a} \times \mathbf{b} = \mathbf{j} - \mathbf{k}$, then $\mathbf{b}$ is equal to
(a) 2i (b) i
(c) 2i − j (d) 2i − k

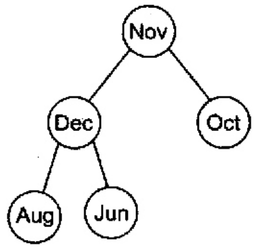
8. The shortest distance of (0, 0) from the curve $y = \dfrac{e^x + e^{-x}}{2}$ is
(a) 1/2
(b) 1
(c) 2
(d) None of the above

9. In an arithmetic progression, the first term is 2, the last term is 29 and the sum is 155. The difference is
(a) 3 (b) 5
(c) 4 (d) 2

10. The AVL Tree (Height Balanced Tree) corresponding to the input sequence "Jun, Aug, Dec, Nov, Oct" is

(a)


(b)


(c)


(d) None of the above

11. In an undirected complete graph with 5 vertices, the number of edges must be equal to
(a) 5 (b) 10
(c) 20 (d) 15

12. Let $f: R \to R$ and $f(x) = \log_e x$, R being the set of real numbers, then
(a) f is onto (b) f is one-one
(c) f is invertible (d) None of these

13. Assume that either $|z| = 1$ or $|w| = 1$ and $\bar{z}w \neq 1$, where z, ω are complex numbers and $\bar{z}$ is the conjugate of z. The value of $\left|\frac{z-w}{1-\bar{z}w}\right|$ is
(a) $\sqrt{2}$ (b) $\sqrt{3}$
(c) $\sqrt{3/2}$ (d) None of these

14. The number of solutions of $10^{2/x} + 25^{1/x} = \left(\frac{65}{8}\right)(50^{1/x})$ is
(a) zero (b) four
(c) two (d) infinite

15. The quadratic equation whose roots are reciprocal of the roots of the equation $x^3 - 3x + 2 = 0$ is
(a) $3x^2 - 2x + 1 = 0$ (b) $2x^2 - x - 1 = 0$
(c) $x^2 - 3x + 2 = 0$ (d) None of these

16. A student is allowed to select at the most n books from a collection of $(2n + 1)$ books. If the total number of ways in which he can select a book is 63, the value of n is
(a) 1 (b) 7
(c) 5 (d) 3

17. The sum of the series $1 + \frac{5}{3} + \frac{5}{3}\cdot\frac{7}{6} + \frac{5}{3}\cdot\frac{7}{6}\cdot\frac{9}{8}\ldots$ is equal to
(a) $3\sqrt{2}$ (b) $9\sqrt{3}$
(c) $5\sqrt{7}$ (d) None of these

18. The value of $\frac{\log_2 24}{\log_{96} 2} - \frac{\log_2 192}{\log_{12} 2}$ is
(a) 0 (b) 1 (c) 2 (d) 3

19. If $P(n)$ is the statement "$n(n+1)(n+2)$ is divisible by 12", then which of the following is not true?
(a) $P(2)$ (b) $P(3)$ (c) $P(4)$ (d) $P(5)$

20. Let f be a function defined on [0, 2], then the function $g(x) = f(9x^2 - 1)$ has domain
(a) [0, 2] (b) [− 1/3, 1/3]
(c) [− 3, 3] (d) None of these

21. If a, b, c are the sides of a triangle, then the value of the expression $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$ is equal to
(a) 1 (b) 3/2 (c) 2 (d) 5/2

22. The number of distinct terms in the expansion of $(x^1 + x^2 + x^3 + \ldots + x^n)^3$ is
(a) ${}^{n+1}C_3$ (b) ${}^{n+2}C_3$ (c) ${}^{n+3}C_3$ (d) ${}^{n+4}C_3$

23. The sum of the series $\frac{\cos\theta}{4} + \frac{\cos 2\theta}{4^2} + \frac{\cos 3\theta}{4^3} + \ldots$ will be equal to
(a) $\frac{4\cos\theta - 1}{17 - 8\cos\theta}$ (b) $\frac{2\cos\theta + 4}{19 - \cos\theta}$
(c) $\frac{\cos\theta}{2 + \cos\theta}$ (d) None of these

24. The number of real solutions of the equation $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{(x^2+x+1)} = \pi/2$ is
(a) zero (b) one
(c) two (d) infinite

25. Let the two statements
I. sin 100 sin 560 sin 700 = 1/8
II. If $\frac{\cos\theta}{a} = \frac{\sin\theta}{b}$ then $\frac{a}{\sec 2\theta} + \frac{b}{\text{cosec}\, 2\theta} = a$
Which of the following, identify the correct statement?
(a) Both I and II are true
(b) Both I and II are false
(c) I is true but II is false
(d) I is false but II is true

26. The number of solutions of the equation $\tan x + \sec x = 2\cos x$, lying in the interval $[0, 2\pi]$ is
(a) 0 (b) 1
(c) 2 (d) 3

27. The statement $p \to q$ is equivalent to
(a) $\sim p$ conjunction q (b) $\sim p$ disjunction q
(c) p conjunction $\sim q$ (d) p disjunction $\sim q$

28. If for a ΔABC, $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$, then $\sin^3 A + \sin^3 B + \sin^3 C$ is equal to
(a) $\sin A + \sin B + \sin C$
(b) $3 \sin A \sin B \sin C$
(c) $\sin 3A + \sin 3B + \sin 3C$
(d) $\sin^3 A \sin^3 B \sin^3 C$

29. A ship arrives at a port and 40 sailors on board go ashore for revelry. Later at night, the 40 sailors return to the ship and in their state of inebriation, each chooses a random cabin to sleep in. What is the expected number of sailors sleeping in their own cabin?
(a) 1/40 (b) 1/4
(c) $(1/40)^{10}$ (d) None of these

30. The complete solution of the equation $7\cos^2 x + \sin x \cos x - 3 = 0$ is given by
(a) $n\pi + \pi/2\ (n \in I)$
(b) $n\pi - \pi/2\ (n \in I)$
(c) $n\pi + \tan^{-1}(4/3)\ (n \in I)$
(d) $n\pi + 3\pi/4,\ k\pi + \tan^{-1}(4/3)\ (n, k \in I)$

31. The choice of throwing 12 in a single throw with three dice is
(a) 12/216 (b) 21/216
(c) 15/216 (d) 25/216

32. If $\int_0^{\pi/2} \frac{dx}{5 + 3\sin x} = \lambda \tan^{-1}\left(\frac{1}{2}\right)$, then the value of λ is given by
(a) 1 (b) 1/2
(c) 1/3 (d) 1/4

33. Using hamming code error detection with the information bit size of five bits the size of the total message using parity bits becomes
(a) 6 (b) 7
(c) 8 (d) 9

34. Suppose, there are 15 different types of coupons and suppose that each time one obtains a coupon, it is equally likely to be any one of the 15 types. The expected number of different types that are contained in the set of 5 coupons is

(a) $15\left[1-\left(\frac{14}{15}\right)^5\right]$ (b) $15\left[1-\left(\frac{14}{15}\right)\right]^{5}$

(c) $5\left[1-\left(\frac{14}{15}\right)\right]^{5}$ (d) None of these

35. The joint density of X and Y is given by
$$f(x, y) = \begin{cases} \frac{1}{2} ye^{-xy}, & 0 < x < \infty, 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

The value of $E[e^{X/2} \mid Y = 1]$ is equal to

(a) e^{-x} (b) 1
(c) 2 (d) e^x

36. The degree and order of differential equation $\sqrt{2\left(\frac{dy}{dx}\right)^3 + 4} = \left(\frac{d^2y}{dx^2}\right)^{3/2}$ are respectively

(a) order 2, degree 3 (b) order 1, degree 3
(c) order 3, degree 2 (d) order 3, degree 1

37. If the sum of the areas of a cube and a sphere is constant, the ratio of an edge of the cube to the diameter of the sphere, when the sum of their volume is minimum, is

(a) 1 : 2 (b) 2 : 1
(c) 1 : 3 (d) None of these

38. The orthocentre of the triangle with vertices (0, 0), (3, 0), (0, 4) is

(a) (0, 0) (b) (3/2, 2)
(c) (1, 4/3) (d) None of these

39. How many ROM bits would be required to build an 8-bit adder/subtractor with mode control, carry input, carry output and two's complement overflow output?

(a) $2^{12} * 10$ (b) $2^{14} * 10$
(c) $2^{16} * 10$ (d) $2^{18} * 10$

40. A straight line is drawn through the centre O of the circle $x^2 + y^2 = 2ax$ parallel to $x + 2y = 0$ and intersecting the circle at A and B. The area of the ΔAOB is

(a) $\frac{a^2}{\sqrt{5}}$ sq units (b) $\frac{a^3}{\sqrt{5}}$ sq units
(c) $\frac{a^2}{\sqrt{3}}$ sq units (d) $\frac{a^2}{\sqrt{2}}$ sq units

41. The area of the portion of the circle $x^2 + y^2 - 4y = 0$ lying below the x-axis is

(a) 24π (b) 42π
(c) 82π (d) 0

42. If $ax + hy + gz = 0$, $hx + by + fz = 0$, $gx + fy + cz = 0$, then

(a) $\frac{x^2}{bc - f^2} = \frac{y^2}{ca - g^2} = \frac{z^2}{ab - h^2}$

(b) $(bc - f^2)(ca - g^2)(ab - h^2) = (fg - ch)(gh - af)(hf - bg)$

(c) $(bc - f^2)(ca - g^2)(ab - h^2) = (fg + ch)(gh + af)(hf + bg)$

(d) $(bc + f^2)(ca + g^2)(ab + h^2) = (fg - ch)(gh - af)(hf - bg)$

43. Which one of the following is an invalid state in an 8421 counter?

(a) 1100 (b) 0010
(c) 0101 (d) 1000

44. The parametric equations $x = \frac{a}{2}(\lambda + 1/\lambda)$; $y = \frac{b}{2}(\lambda - 1/\lambda)$ where λ is a parameter, represents

(a) a straight line (b) a parabola
(c) an ellipse (d) a hyperbola

45. If two forces act at a given point, the resultant of these forces can never have

(a) the magnitude of either of these forces
(b) the direction of either of these forces
(c) a magnitude that is less than that of either of these forces
(d) a magnitude that is greater than the algebraic sum of these forces

46. To implement the expression $\bar{A}BCD + ABCD + \overline{ABC}D$, is take one OR gate and

(a) one AND gate
(b) three AND gates
(c) three AND gates and four inverters
(d) three AND gates and three inverters

47. If the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide, then the value of b^2 is

(a) 3 (b) 16
(c) 9 (d) 12

48. What will the effect of the following program in *C*?

```
# include < stdio.h>
main ( )
{
int a[10], *p;
p = a;
a[0] = 1;
a[1] = 2;
(*p) ++;
return (0);
}
```

(a) Value of a[0] will be 1 and a[1] will be 2
(b) Value of a[0] will be 2 and a[1] will be 2
(c) Value of a[0] will be 1 and a[1] will be 3
(d) Value of a[0] will be 3 and a[1] will be 3

49. If a line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube, then $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta$ is equal to

(a) 1/3 (b) 2/3
(c) 4/3 (d) 8/3

50. A *J-K* flip-flop with $J = 1$ and $K = 1$ has a 10 kHz clock input. The output is

(a) constantly high
(b) constantly low
(c) a 10 kHz square wave
(d) a 5 kHz square wave

51. Equation of a common tangent to the curves $y^2 = 8x$ and $xy = -1$ is
(a) $3y = 9x + 2$ (b) $y = 2x + 1$
(c) $2y = x + 8$ (d) $y = x + 2$

52. In the microprocessor 8085, the temporary register holds
(a) temporary results during program execution
(b) next address for execution
(c) one of the operands needed for execution
(d) status information

53. The greatest distance of point $P(10, 7)$ from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ is
(a) 10 (b) 15
(c) 5 (d) 20

54. Let $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{c} = \mathbf{i} + \mathbf{j} - \mathbf{k}$. A vector in the plane of **a** and **b** whose projection on **c** is $1/\sqrt{3}$, is
(a) $4\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ (b) $2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$
(c) $3\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ (d) $4\mathbf{i} + \mathbf{j} - 4\mathbf{k}$

55. On a Karnaugh map, grouping the 0's produces
(a) a POS expression
(b) an SOP expression
(c) AND-OR logic
(d) a 'don't care' condition

56. The value of p such that the unit vectors $\mathbf{a} = \dfrac{2\mathbf{i} + p\mathbf{j} + \mathbf{k}}{\sqrt{5 + p^2}}$ and $\mathbf{b} = \dfrac{\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}}{\sqrt{14}}$ are orthogonal is
(a) 2/5 (b) 5/2
(c) 3/7 (d) 2/7

57. If A and B are coefficients of x^n in the expansions of $(1 + x)^{2n}$ and $(1 + x)^{2n-1}$ respectively, then A/B is equal to
(a) 1 (b) 2
(c) 1/2 (d) $1/n$

58. Assuming that ϕ is the angle of friction, the least force which moves a weight W along a rough horizontal plane is
(a) $W \sin \phi$ (b) $W \cos \phi$
(c) $W \tan \phi$ (d) None of these

59. The centre of gravity of the volume formed by the revolution of the portion of the parabola $y^2 = 4ax$, cut off by the ordinate $x = h$ along x-axis is
(a) $\bar{x} = \frac{1}{4} h, \bar{y} = 0$ (b) $\bar{x} = \frac{3}{4} h, \bar{y} = 0$
(c) $\bar{x} = 0, \bar{y} = \frac{2}{3} h$ (d) $\bar{x} = \frac{2}{3} h, \bar{y} = 0$

60. With every bit added the magnitude of the number
(a) increases by 2 (b) decreases by 2
(c) multiplies by 2 (d) divides by 2

61. A uniform chain of length L and mass M is lying on a smooth table and one-third of its length is hanging vertically down over the edge of the table. If g is the acceleration due to gravity, the work required to pull the hanging part on the table is
(a) MgL (b) $MgL/3$
(c) $MgL/9$ (d) $MgL/18$

62. $\int_0^1 \dfrac{3x}{(x+1)(x-2)}$ is given by
(a) log 2 (b) 1/3 log 2
(c) 1/2 log 2 (d) log 1/2

63. Is it possible for the processes to complete their execution without entering deadlock in any unsafe system?
(a) Not possible
(b) Possible but not permissible
(c) Possible and permissible
(d) None of the above

64. In the rational Q, the sequence $1, -2, 3, -4, 5, -6, \ldots$ is
(a) a cauchy sequence
(b) bounded
(c) convergent
(d) None of the above

65. Let A and B be non-empty subsets of real line R. Which of the following statements would be equivalent to $\sup(A) \le \inf(B)$?
(a) For every a in A there exists a b in B such that $a \le b$
(b) There exists a in A and b in B such that $a \le b$
(c) For every a in A and every b in B, we have $a \le b$
(d) There exists a in A such that $a \le b$ for all b in B

66. In a buddy memory management, memory is divided into
(a) fixed size frames
(b) variable size frames
(c) frame size in some power of 2
(d) continuous allocation

67. Let $f : (1, 5) \to R$ be a uniformly continuous function such that $f(2) = 3$ and $f(4) = 6$. The most we can say about the set $f(1, 5)$ is that
(a) it is a set which contains [3, 6]
(b) it is a bounded set which contains [3, 6]
(c) it is an open interval
(d) it is an interval which contains [3, 6]

68. What expresses the specific number of entity occurrences associated with one occurrence of the related entity?
(a) Connectivity (b) Cardinality
(c) Degree (d) None of these

69. If E is the union of a Borel set and a null set, the best one can say about E is that it is
(a) a Lebesgue measurable set
(b) a Borel set
(c) an arbitrary set
(d) a G-delta set

70. If $x : y = 3 : 4$, then the ratio of $7x - 4y : 3x + y$ is
(a) 5/18 (b) 13/18
(c) 5/13 (d) None of these

71. If y varies as the sum of two quantities of which one varies directly as x and other varies inversely and if $y = 6$ when $x = 4$, and $y = 10/3$ when $x = 3$; the equation between x and y is
(a) $y = x - 4/x$ (b) $y = 2x - 4/x$
(c) $y = x - 8/x$ (d) None of these

72. If $\sqrt{\log_2 x} - 0.5 = \log_2 \sqrt{x}$, then, x equals
(a) 4 (b) 8
(c) 16 (d) None of these

73. In a paged memory system, with page size of 1 K and logical address of 18 bits, how many pages are possible?
(a) 512 (b) 1024
(c) 256 (d) 2096

74. The solution of the equation $(2x+y+1)dx+(4x+2y-1)\,dy=0$ is
(a) $\log(2x+y-1)=C+x+y$
(b) $\log(4x+2y-1)=C+2x+y$
(c) $\log(2x+y+1)+x+2y=C$
(d) $\log(2x+y-1)+x+2y=C$

75. The value of $\sin(2\tan^{-1}(1/3))+\cos(\tan^{-1}2\sqrt{2})$ is
(a) 12/13 (b) 13/14
(c) 14/15 (d) None of these

76. A relation R is said to be partial order, if
(a) R is reflexive, symmetric and transitive
(b) R is reflexive, asymmetric and transitive
(c) R is reflexive, antisymmetric and transitive
(d) R is reflexive, antisymmetric but not transitive

77. The angle of the elevation of the sum when the length of the shadow of the pole is $\sqrt{3}$ times, the height of the pole is
(a) 30° (b) 45°
(c) 60° (d) 135°

78. If the ratio of sum of m terms to the sum of n terms in an AP is m^2 to n^2, then the mth term to the nth term is
(a) $m-1:n-1$ (b) $2m+1:2n+1$
(c) $2m-1:2n-1$ (d) None of these

79. The sum of infinite series $1+3x+6x^2+10x^3+\ldots, x<1$ is
(a) $\frac{1}{1-3x}$ (b) $\frac{1}{1-x^3}$
(c) $\frac{1}{1-x^2}$ (d) $\frac{1}{(1-x)^3}$

80. If $f(x)=\begin{vmatrix}\sin^2\theta & \cos^2\theta & x\\ \cos^2\theta & x & \sin^2\theta\\ x & \sin^2\theta & \cos^2\theta\end{vmatrix}$ $\theta\in(0,\pi/2)$, then roots of $f(x)=0$ are
(a) $1/2, -1$ (b) $1/2, -1, 0$
(c) $-1/2, 1, 0$ (d) $-1/2, -1, 0$

81. For the C program given below, what will be the output?

```
# include <stdio.h>
main ( )
{
char ar [ ] = "computer science"
print f("\n%c", *(ar++));
return (0);
}
```

(a) c
(b) 0
(c) Compile-time error message
(d) Run-time error message

82. A speaks truth 3 times out of 4 and B speaks 7 times out of 10. They both assert that a white ball has been drawn from a bag containing 6 different colour balls. Find the probability of the truth of the assertion.
(a) 21/40 (b) 35/36
(c) 39/40 (d) None of these

83. The roots of the equation $6x^{3/4}=7x^{1/4}-2x^{-1/4}$ are
(a) 4/9 and 1/9 (b) 9/4 and 1/4
(c) 4/9 and 1/4 (d) None of these

84. The three sides of a trapezium are equal and each being 6 cm long. The maximum area of trapezium is
(a) $27\sqrt{3}$ cm^2 (b) $36\sqrt{3}$ cm^2
(c) $72\sqrt{3}$ cm^2 (d) None of these

85. For a machine with 48-bit virtual address, 32-bit physical address and page size 8 K, how many entries will be there in an inverted page table?
(a) 2^{32} (b) 2^{48} (c) 2^{13} (d) 2^{19}

86. If $\sin(x+3\alpha)=3\sin(\alpha-x)$, then
(a) $\tan x=\tan\alpha$ (b) $\tan x=\tan^2\alpha$
(c) $\tan x=\tan^3\alpha$ (d) $\tan x=3\tan\alpha$

87. The rule of inference stated as $a\to b, b\to c \vdash a\to c$ is known as
(a) modus ponens (b) modus tollens
(c) syllogism (d) None of these

88. The solution of the differential equation $y(x^2y+e^x)dx-e^xdy=0$ is
(a) $3xy^3+e^x=Cx$ (b) $xy^3+3e^x=Cx$
(c) $3x^3y^3+e^x=Cx$ (d) $x^3y+3e^x=Cy$

89. The function f defined by $f(x)=x[1+1/3\sin(\log x^2)]$, $x\neq 0$, $f(0)=0$ ([] represents the greatest integer function) is
(a) continuous and differentiable at origin
(b) not continuous but differentiable
(c) continuous but not differentiable
(d) not continuous and not differentiable

90. If f is twice differentiable function such that $f''(x)=-f(x)$, $f'(x)=g(x)$ and $h(x)=[f(x)]^2+[g(x)]^2$, also if $h(5)=11$, then $h(10)$ is equal to
(a) 22 (b) 121
(c) 16 (d) None of these

91. $\tan A+2\tan 2A+4\tan 4A+8\cot 8A$ is equal to
(a) $\tan 2A$ (b) $\cot A$
(c) $\sin 3A$ (d) None of these

92. The isolated mapped $\underline{IO}$ scheme resulted in introduction of one more pin M/IO. It helps in
(a) isolating memory with the IO devices
(b) increasing the number of addressable memory locations
(c) increasing the number of IO devices
(d) All of the above

93. In four throws with a pair of dice, what is the chance of throwing doublets atleast twice?
(a) 1/144 (b) 25/144 (c) 19/144 (d) 26/144

94. If $x=\cos\theta$, $y=\sin^3\theta$, then which of the following equations is true?
(a) $\frac{d^2y}{dx^2}+y\frac{dy}{dx}=\sin^2\theta\,(5\cos^2\theta-1)$
(b) $y\frac{d^2y}{dx^2}+\frac{dy}{dx}=3\sin^2\theta\,(5\cos^2\theta-1)$
(c) $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=3\sin^2\theta\,(5\cos^2\theta-1)$
(d) None of the above

95. If a file of size $n = 1000$ takes, on an average, 3 ms for searching an item using binary search algorithm, then approximately, how much time, on an average, would it take to search an item in a file of size $n = 1000000000$?
(a) 9000000 ms (b) 9 ms
(c) 3000000 ms (d) None of these

96. A disk of 30 MByte capacity uses block size of 512 bytes and 4 blocks/clusters. How many entries are required in FAT (File Allocation Table)?
(a) 30 K (b) 512 K (c) 15 K (d) 60 K

97. The value of $\int \frac{x}{\sec x + 1}\, dx$ is
(a) $\frac{x}{2} - \frac{x^2}{2} \tan\left(\frac{x}{2}\right) + 2 \log (\sec x)$
(b) $\frac{x^2}{2} - x \tan\left(\frac{x}{2}\right) + 2 \log (\sec x/2)$
(c) $\frac{x^2}{2} - \frac{x^2}{2} \tan\left(\frac{x}{2}\right) + 2 \log (\sec x)$
(d) None of the above

98. An entity and its subclasses and their subclasses and so on are called as
(a) type hierarchy (b) range hierarchy
(c) inheritance (d) None of these

99. The value of $\int \frac{x - \sin x}{1 - \cos x}\, dx$ is
(a) $-\frac{x^2}{2} \cot\left(\frac{x}{2}\right)$ (b) $\frac{x^2}{2} \cot\left(\frac{x}{2}\right)$
(c) $-x \cot\left(\frac{x}{2}\right)$ (d) $x \cot\left(\frac{x}{2}\right)$

100. The limit of $A^x \sin\left(\frac{B}{A^x}\right)$ where $x \to \infty$ and $0 < A < 1$ is
(a) B (b) 1
(c) A (d) 0

101. The solution of the equation $\frac{dy}{dx} = \frac{3x - 4y - 2}{3x - 4y - 3}$ is
(a) $(x - y)^2 + C = \log (3x - 4y + 1)$
(b) $x - y + C = \log (3x - 4y + 4)$
(c) $x - y + C = \log (3x - 4y - 3)$
(d) $x - y + C = \log (3x - 4y + 1)$

102. Which of the following C statements is not correct?
(a) $a = a + 1$ (b) $a + = 1$ (c) $a + +$ (d) $a = + 1$

103. A particle is moving Eastwards with velocity 5 m/s. In 10 s, the velocity changes to 5 m/s Northwards. The average acceleration in this time is
(a) zero
(b) $\frac{1}{\sqrt{2}}$ m / s^2 towards North-West
(c) $\frac{1}{\sqrt{2}}$ m / s^2 towards North-East
(d) $\frac{1}{2}$ m / s^2 towards North

104. Which integrity constraint guarantees that every primary key attribute is non-null?
(a) Domain integrity (b) Key integrity
(c) Entity integrity (d) Referential integrity

105. The area of the region bounded by the curves $x^2 + y^2 = a^2$ and $x + y = a$ in the first quadrant is given by
(a) $\int_0^a \int_{a-x}^{\sqrt{a^2 - x^2}} dx dy$ (b) $\int_0^a \int_{a-x}^{\sqrt{a^2 - x^2}} dy dx$
(c) $\int_{a-x}^{\sqrt{a^2 - x^2}} \int_0^a dx dy$ (d) None of these

106. Which of the following rules states that every piece of data in a relational database, can be accessed by using a combination of a table name, a primary key value that identifies the row and the column name, which identifies the cell?
(a) Information rule
(b) Non-subversion rule
(c) Integrity rule
(d) Guaranteed access rule

107. If $A = \begin{bmatrix} 5 & 0 & -2 \\ 0 & 1 & 0 \\ -4 & 0 & -1 \end{bmatrix}$ and I be 3×3 unit matrix, then rank of $I - A$ is
(a) 0 (b) 1
(c) 2 (d) 3

108. The program that combines the output of compiler with various library functions to produce an executable image is called
(a) loader (b) linker
(c) assembler (d) debugger

109. Which of the following is false?
(a) If A is a square matrix, then Adj $A' = (\text{Adj } A)'$
(b) If I is the identity matrix of order n, then Adj $I = I$
(c) $(A^*)^{-1} = (A^{-1})^*$
(d) If A and B are invertible, then $AB = BA$

110. If ABC is not a right triangle, then the value of
$$\Delta = \begin{vmatrix} \tan A & 1 & 1 \\ 1 & \tan B & 1 \\ 1 & 1 & \tan C \end{vmatrix}$$ is
(a) -1 (b) 2
(c) 3 (d) 0

111. If the expression $((2 + 3) \times 4 + 5 \times (6 + 7) \times 8) + 9$ is evaluated with $\times$ having precedence over $+$, then the value obtained in prefix notation for the expression will be
(a) $+ \times + + 234 \times \times 5 + 6789$
(b) $\times + + 234 \times \times 5 + + 6789$
(c) $+ \times + \times 234 + + 5 \times 6789$
(d) None of the above

112. If $e^{ix} = \cos x + i \sin x$ and
$$x + iy = \begin{vmatrix} 1 & e^{\pi i/4} & e^{\pi i/3} \\ e^{-\pi i/4} & 1 & e^{2\pi i/3} \\ e^{-\pi i/3} & e^{-2\pi i/3} & e^{-2\pi i} \end{vmatrix}$$ then
(a) $x = -1, y = \sqrt{2}$ (b) $x = 1, y = -\sqrt{2}$
(c) $x = -\sqrt{2}, y = \sqrt{2}$ (d) None of these

113. The determinant $\begin{vmatrix} 1 & 1+i & i \\ 1+i & i & 1 \\ i & 1 & 1+i \end{vmatrix}$ equals

(a) $7+4i$ (b) $2-2i$
(c) $-7-4i$ (d) $-2+2i$

114. Given an n-bit number, we can represent two's complement numbers in the range

(a) 0 to 2^{n-1} (b) 0 to $2^{n-1}-1$
(c) -2^{n-1} to 2^{n-1} (d) -2^{n-1} to $2^{n-1}-1$

115. The solution of $y^5x + y - x\dfrac{dy}{dx} = 0$ is

(a) $x^4/4 + 1/5(x/y)^5 = C$
(b) $x^5/5 + 1/4(x/y)^4 = C$
(c) $(x/y)^5 + x^4/4 = C$
(d) $(xy)^4 + x^5/5 = C$

116. Which of the following sub-query is resolved in the top to bottom fashion?

(a) Nested
(b) Parallel
(c) Correlated
(d) None of the above

117. The number of irrational solutions of the equation $\sqrt{x^2+\sqrt{x^2+11}} + \sqrt{x^2-\sqrt{x^2+11}} = 4$ is

(a) 0 (b) 2
(c) 4 (d) indefinite

118. The term independent of x in the expansion of $(2x^2 - 1/x)^{12}$ is

(a) 7910 (b) 7920
(c) 7930 (d) 7900

119. A and B throw with one die for a stake of ₹ 11 which is to be won by the player who first throws 6. If A has the first throw, what is the expectation of his winning the stake?

(a) 5/11 (b) 15/26
(c) 6/11 (d) 16/26

120. A river is flowing from West to East at a speed of 5 m/min. A man on the South bank of the river, capable of swimming at 10 m/min in still water, wants to swim across the river in the shortest time. He should swim in a direction

(a) towards North
(b) 30° East of North
(c) 30° West of North
(d) 60° East of North

Answers *with* Solutions

1. (c) $f_1(z) = \begin{cases} \dfrac{(R_e(z^2))^2}{|z|^2}, & \text{if } z \neq 0 \\ 0, & \text{if } z = 0 \end{cases}$

Let $z = x + iy$

$\Rightarrow f_1(z) = \begin{cases} \dfrac{(x^2-y^2)^2}{x^2+y^2}, & \text{if } z \neq 0 \\ 0, & \text{if } z = 0 \end{cases}$

By taking limit along $y = mx$, we get

$$\lim_{x\to 0} \frac{(x^2 - m^2x^2)^2}{x^2(1+m^2)} = \lim_{x\to 0} \frac{x^2(1-m^2)^2}{1+m^2} = 0$$

which is independent of m.

So, $\lim_{z\to 0} f_1(z) = 0 = f_1(0)$

Hence, $f_1(z)$ is continuous at $z = 0$ ...(i)

$f_2(z) = |z|^2 = x^2 + y^2 = u + iv$ (say)

$\Rightarrow u = x^2 + y^2; v = 0$

$\Rightarrow \dfrac{\partial u}{\partial x} = 2x; \dfrac{\partial u}{\partial y} = 2y; \dfrac{\partial v}{\partial x} = \dfrac{\partial v}{\partial y} = 0$

Cauchy-Riemann equation

$\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}; \dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$ is satisfied at the origin

$$f'(0) = \lim_{z\to 0} \frac{f(z) - f(0)}{z} = \lim_{z\to 0} \frac{|z|^2 - 0}{z} = \lim_{z\to 0} \frac{x^2+y^2}{x+iy}$$

Let $z \to 0$ along the line $y = mx$, then

$$f'(0) = \lim_{x\to 0} \frac{x^2 + m^2x^2}{x + imx} = \lim_{x\to 0} \frac{x(1+m^2)}{1+im} = 0$$

$\Rightarrow f'(0)$ is unique.

Hence, $f_2(z) = |z|^2$ is differentiable at $z = 0$

$\Rightarrow f_2(z) = |z|^2$ is analytic at $z = 0$

$\therefore$ Both I and II are true.

2. (c) Probability density function of X which is uniformly distributed in (0, 10) is

$$f(x) = \begin{cases} \dfrac{1}{10}, & \text{if } 0 < x < 10 \\ 0, & \text{otherwise} \end{cases}$$

$$\therefore \quad P(1 < X < 6) = \int_1^6 \frac{1}{10}\,dx = \left[\frac{x}{10}\right]_1^6 = \frac{5}{10}$$

3. (c) Area of triangle with vertices $(p, q+r), (q, p+r), (r, p+q)$ is

$$A = \frac{1}{2}\begin{vmatrix} p & q+r & 1 \\ q & p+r & 1 \\ r & p+q & 1 \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2$

$$= \frac{1}{2}\begin{vmatrix} p+q+r & q+r & 1 \\ q+p+r & p+r & 1 \\ r+p+q & p+q & 1 \end{vmatrix}$$

$$= \frac{1}{2}(p+q+r)\begin{vmatrix} 1 & q+r & 1 \\ 1 & p+r & 1 \\ 1 & p+q & 1 \end{vmatrix} = 0$$

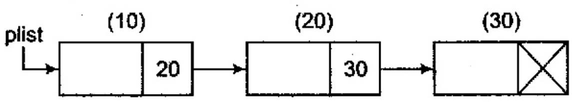
4. (b) Equation of plane is

$$\frac{x}{20} + \frac{y}{15} + \frac{z}{-12} = 1$$

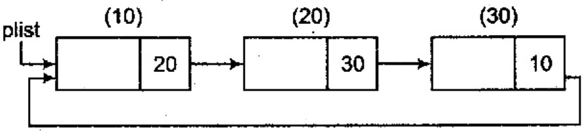
$\therefore$ Volume of tetrahedron

$$V = \frac{1}{6} \times 20 \times 15 \times 12 = 600 \text{ cu units}$$

5. (a) List will become circular. Because plist is a pointer type variable, so it contains the address of first node.



$\therefore$ plist = 10
Now, temp = 10
and loop will go on until temp → link becomes null. So, finally temp = 30 and check (temp → link) that is null so loop is ended. Now, (temp → link) will contain the address that is holded by plist pointer.
$\therefore$ (temp → link) = 10
So, now list will be as follows



So, list will become circular.

6. (d) There is a mistake in the question is wrong as the point (1, 2) lies outside the parabola

$$y^2 = x$$

7. (b) Let $\mathbf{b} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$; $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$

$$\mathbf{a} \cdot \mathbf{b} = x + y + z = 1 \qquad \dots(i)$$

$$\therefore \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = (z - y)\mathbf{i} + (x - z)\mathbf{j} + (y - x)\mathbf{k}$$

$$\mathbf{a} \times \mathbf{b} = \mathbf{j} - \mathbf{k}$$

$\Rightarrow$ $z - y = 0 \Rightarrow y = z$
$x - z = 1 \Rightarrow x = z + 1$
$y - x = -1$

Putting in Eq. (i), we get
$z + 1 + z + z = 1 \Rightarrow 3z = 0 \Rightarrow z = 0$
$\Rightarrow$ $x = 1, y = 0$
Hence, $\mathbf{b} = \mathbf{i}$

8. (b) Any point on the curve is $P\left(x, \frac{e^x + e^{-x}}{2}\right)$

Let $d = OP$

$$\Rightarrow d^2 = OP^2 = x^2 + \left(\frac{e^x + e^{-x}}{2}\right)^2$$

which is minimum at $x = 0$
$\therefore$ Minimum value of $d = 1$

9. (a) Let d be the common difference and the last term be the nth term.

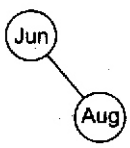
$\Rightarrow$ $T_n = a + (n-1)d$

and $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow$ $29 = 2 + (n-1)d \Rightarrow (n-1)d = 27$

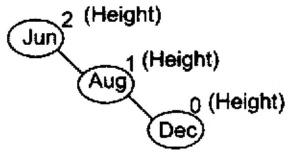
and $155 = \frac{n}{2}(2 + 29) \Rightarrow n = 10 \Rightarrow 9d = 27 \Rightarrow d = 3$

10. (d) An AVL tree (Height Balanced tree) must also be a Binary Search Tree (BST)
Input sequence : Jun, Aug, Dec, Nov, Oct.
Insert Jun-Jun
Insert Aug

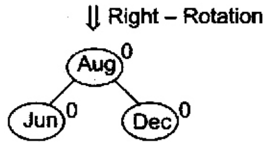


($\because$ Jun comes before Aug)

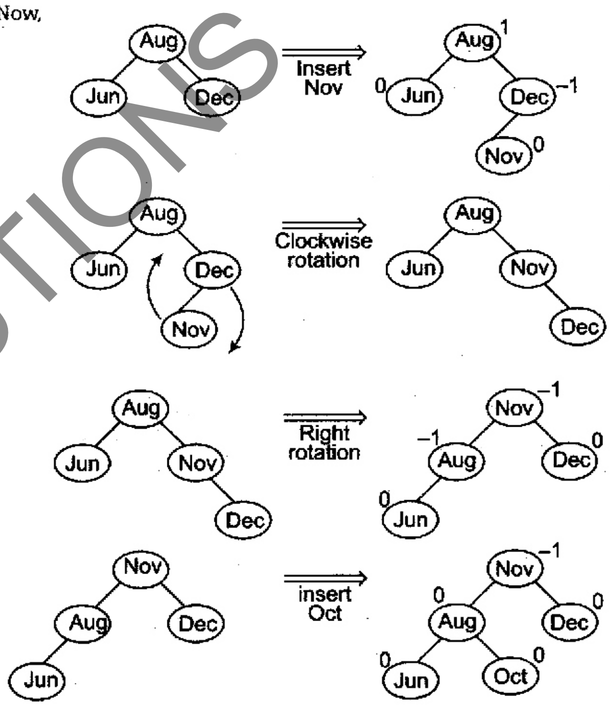
Insert Dec



Height of tree = (height of right subtree − height of left subtree)



Now,



So, option (d) is correct.

11. (b) In an undirected complete graph, if there are n vertices, then the number of edges are

$$e = \frac{n(n-1)}{2}$$

Here, $n = 5$

$$e = \frac{5 \times 4}{2}$$

$$e = 10$$

12. (d) $f(x) = \log_e x$ is not a function from R to R as logarithm of non-positive values does not exist.

13. (d) If $|z| = 1$, then $|\bar{z}| = 1$ and $z\bar{z} = |z|^2 = 1$

$$\Rightarrow \left|\frac{z - w}{1 - \bar{z}w}\right| = \frac{|z - w|}{|-\bar{z}w|} = \frac{|z - w|}{|z\bar{z} - \bar{z}w|}$$

$$= \frac{|z - w|}{|\bar{z}||z - w|} = \frac{1}{|\bar{z}|} = 1$$

14. (c) $$(10)^{\frac{2}{x}} + (25)^{\frac{1}{x}} = \left(\frac{65}{8}\right)(50^{\frac{1}{x}})$$

$$\Rightarrow (100)^{\frac{1}{x}} + (25)^{\frac{1}{x}} = \left(\frac{65}{8}\right)(50^{\frac{1}{x}})$$

$\Rightarrow \quad (4)^{\frac{1}{x}} + 1 = \left(\frac{65}{8}\right)(2^{\frac{1}{x}})$

$\Rightarrow \quad 8y^2 + 8 = 65y \qquad \left(\text{let } 2^{\frac{1}{x}} = y\right)$

$\Rightarrow \quad 8y^2 - 65y + 8 = 0$

$\Rightarrow \quad (8y-1)(y-8) = 0$

$\Rightarrow \quad y = \frac{1}{8}, 8$

$\Rightarrow \quad 2^{\frac{1}{x}} = \left(\frac{1}{2}\right)^3, 2^3$

$\Rightarrow \quad 2^{\frac{1}{x}} = 2^{-3}, 2^3$

$\Rightarrow \quad \frac{1}{x} = -3, 3$

$\Rightarrow \quad x = -\frac{1}{3}, \frac{1}{3}$

i.e., two solutions are possible.

15. (b) $x^3 - 3x + 2 = 0$

$\Rightarrow (x+2)(x-1)^2 \Rightarrow x = 1, 1, -2$

Reciprocal of these roots are 1 and $-\frac{1}{2}$.

So, quadratic equation is $(x-1)(x+\frac{1}{2}) = 0$

$\Rightarrow \quad (x-1)(2x+1) = 0 \Rightarrow 2x^2 - x - 1 = 0$

16. (d) Number of ways in which at most n books from $(2n+1)$ books will be selected is

$x = {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots\ldots + {}^{2n+1}C_n = 63$

Since, $\quad {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots\ldots + {}^{2n+1}C_n + {}^{2n+1}C_{n+1} + \ldots\ldots + {}^{2n+1}C_{2n+1} = 2^{2n+1}$

$\Rightarrow \quad 2({}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots\ldots + {}^{2n+1}C_n) = 2^{2n+1}$

(by using ${}^nC_r = {}^nC_{n-r}$)

$\Rightarrow \quad 1 + x = 2^{2n} \Rightarrow 2^{2n} = 64 = 2^6$

$\Rightarrow \quad n = 3$

17. (b) If n is a negative integer or fraction, then by using Binomial theorem,

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots\ldots$

Given series is

$S = 1 + \frac{5}{3} + \frac{5}{3}\cdot\frac{7}{6} + \ldots\ldots$

$\Rightarrow \quad nx = \frac{5}{3}; \frac{n(n-1)}{2}x^2 = \frac{5}{3}\left(\frac{7}{6}\right)$

$\Rightarrow \quad nx\left(\frac{nx}{2} - \frac{x}{2}\right) = \frac{5}{3}\left(\frac{5}{6} + \frac{2}{6}\right)$

$\Rightarrow \quad \frac{5}{3}\left(\frac{5}{6} - \frac{x}{2}\right) = \frac{5}{3}\left(\frac{5}{6} - \left(-\frac{1}{3}\right)\right)$

$\Rightarrow \quad \frac{x}{2} = -\frac{1}{3} \Rightarrow x = -\frac{2}{3}$

As, $nx = \frac{5}{3} \Rightarrow n = \frac{-5}{2}$

$\Rightarrow \quad S = \left(1 - \frac{2}{3}\right)^{\frac{-5}{2}} = \left(\frac{1}{3}\right)^{\frac{-5}{2}} = (3)^{\frac{5}{2}} = 9\sqrt{3}$

18. (d) $\frac{\log_2 24}{\log_{96} 2} - \frac{\log_2 192}{\log_{12} 2}$

$= \log_2 24 \log_2 96 - \log_2 192 \log_2 12$

$= (\log_2 2^3 \cdot 3)(\log_2 2^5 \cdot 3) - (\log_2 2^6 \cdot 3)(\log_2 2^2 \cdot 3)$

$= (3 + \log_2 3)(5 + \log_2 3) - (6 + \log_2 3)(2 + \log_2 3)$

$= 3$

19. (d) $P(5) = 5(6)(7) = 210$ is not divisible by 12.

20. (d) $f(x)$ is defined for $0 \le x \le 2$.

$\Rightarrow \quad f(9x^2 - 1)$ is defined for $0 \le 9x^2 - 1 \le 2$.

$\Rightarrow \quad 1 \le 9x^2 \le 3$

$\Rightarrow \quad \frac{1}{9} \le x^2 \le \frac{1}{3}$

$\Rightarrow \quad -\frac{1}{\sqrt{3}} \le x \le -\frac{1}{3}$

or $\quad \frac{1}{3} \le x \le \frac{1}{\sqrt{3}}$

$\Rightarrow \quad x \in \left[-\frac{1}{\sqrt{3}}, -\frac{1}{3}\right] \cup \left[\frac{1}{3}, \frac{1}{\sqrt{3}}\right]$

21. (b) In case of equilateral triangle, $a = b = c$

$\Rightarrow \quad \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$

$= \frac{3}{2}$

22. (b) Number of distinct terms in the expansion of $(x^1 + x^2 + x^3 + \ldots + x^n)^3$ is equal to number of ways of distributing 3 identical items among n persons such that any one can get any number of powers.

$= {}^{3+n-1}C_{n-1} = {}^{n+2}C_{n-1} = {}^{n+2}C_3$

23. (a) $S = \frac{\cos\theta}{4} + \frac{\cos 2\theta}{4^2} + \frac{\cos 3\theta}{4^3} + \ldots$

$= \text{Real part of } \frac{e^{i\theta}}{4} + \frac{e^{i2\theta}}{4^2} + \frac{e^{i3\theta}}{4^3} + \ldots$

$= \text{Re}\left(\frac{e^{i\theta}}{4} + \left(\frac{e^{i\theta}}{4}\right)^2 + \left(\frac{e^{i\theta}}{4}\right)^3 + \ldots\right)$

$= \text{Re}\left(\frac{\frac{e^{i\theta}}{4}}{1 - \frac{e^{i\theta}}{4}}\right)$ (sum of infinite terms of GP)

$= R\left(\frac{\cos\theta + i\sin\theta}{4 - \cos\theta - i\sin\theta}\right)$

$= R\left(\frac{\cos\theta + i\sin\theta}{(4-\cos\theta) - i\sin\theta} \times \frac{4 - \cos\theta + i\sin\theta}{4 - \cos\theta + i\sin\theta}\right)$

$= R\left(\frac{\cos\theta(4-\cos\theta) - \sin^2\theta + i(4\sin\theta)}{(4-\cos\theta)^2 + \sin^2\theta}\right)$

$= \frac{4\cos\theta - 1}{17 - 8\cos\theta}$

24. (c) $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ ...(i)

Let $\quad \tan^{-1}\sqrt{x(x+1)} = \theta$

$\Rightarrow \quad \tan\theta = \sqrt{x(x+1)} = \sqrt{x^2+x}$

$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{x^2+x+1}}$

$\Rightarrow \quad \theta = \cos^{-1}\frac{1}{\sqrt{x^2+x+1}}$

Now, from Eq. (i),

$\cos^{-1}\frac{1}{\sqrt{x^2+x+1}} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$

$\Rightarrow \quad \cos^{-1}\frac{1}{\sqrt{x^2+x+1}} = \frac{\pi}{2} - \sin^{-1}\sqrt{x^2+x+1}$

$= \cos^{-1}\sqrt{x^2+x+1}$

$\Rightarrow \quad \dfrac{1}{\sqrt{x^2+x+1}} = \sqrt{x^2+x+1}$

$\Rightarrow \quad x^2+x+1 = 1 \Rightarrow x^2+x=0$

$\Rightarrow \quad x(x+1)=0$

$\Rightarrow \quad x = 0, -1$

Hence, two solutions exist.

25. (d) I. $\sin 100 \sin 560 \sin 700$

$= \sin(90+10)\sin(360+180+20)\sin(720-20)$

$= \cos 10(-\sin 20)(-\sin 20)$

$= \sin^2 20 \cos 10$

$= \dfrac{\sin^2 20 (2\sin 10 \cos 10)}{2\sin 10}$

$= \dfrac{\sin^3 20}{2\sin 10} = \dfrac{4\sin^3 20}{8\sin 10}$

$= \dfrac{3\sin 20 - \sin 60}{8\sin 10} \neq \dfrac{1}{8}$

II. Let $\dfrac{\cos\theta}{a} = \dfrac{\sin\theta}{b} = k$ (say)

$\Rightarrow \quad \dfrac{1}{a^2+b^2} = k^2$

$\Rightarrow \quad \cos\theta = ak; \sin\theta = bk$

Now, $\cos 2\theta = \cos^2\theta - \sin^2\theta = (a^2-b^2)k^2$

$\sin 2\theta = 2\sin\theta\cos\theta = 2abk^2$

Now, $\dfrac{a}{\sec 2\theta} + \dfrac{b}{\text{cosec} 2\theta}$

$= a\cos 2\theta + b\sin 2\theta$

$= [a(a^2-b^2) + b(2ab)]k^2$

$= a(a^2+b^2)k^2 = a$

So, II is true.

26. (c) $\tan x + \sec x = 2\cos x$

$\Rightarrow \quad \dfrac{\sin x}{\cos x} + \dfrac{1}{\cos x} = 2\cos x$

$\Rightarrow \quad \dfrac{\sin x + 1}{\cos x} = 2\cos x$

$\Rightarrow \quad \sin x + 1 = 2\cos^2 x$

$\Rightarrow \quad 2(1-\sin^2 x) = 1 + \sin x$

$\Rightarrow \quad 2\sin^2 x + \sin x - 1 = 0$

$\Rightarrow \quad (2\sin x - 1)(\sin x + 1) = 0$

$\Rightarrow \quad \sin x = \dfrac{1}{2}$ or $\sin x = -1$

$\Rightarrow \quad x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

or $\quad x = \dfrac{3\pi}{2}$

$x = \dfrac{3\pi}{2}$ cannot be a solution, as at $x = \dfrac{3\pi}{2}$ both $\tan x$ and $\sec x$ does not exist. So, $x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$ *i.e.*, two solutions are there in the interval $[0, 2\pi]$.

27. (b) $p \to q$ is equivalent to $\sim p \vee q$.

Where $\vee$ is also known as disjunction.

28. (b) $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix}$;

$C_1 \to C_1 + C_2 + C_3$

$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$;

Taking $a+b+c$ common from first column.

$= (a+b+c)(bc - a^2 - b^2 + ac + ab - c^2)$

$= -(a^3+b^3+c^3-3abc)$

$\Delta = 0 \Rightarrow a^3+b^3+c^3 = 3abc$ ...(i)

As, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$ ...(ii)

in a triangle.

So, putting values of a, b, c from Eq. (ii) in Eq. (i), we get

$8R^3(\sin^3 A + \sin^3 B + \sin^3 C) = 3(8R^3)\sin A \sin B \sin C$

$\Rightarrow \quad \sin^3 A + \sin^3 B + \sin^3 C = 3\sin A \sin B \sin C$

29. (d) Probability that the sailor returns in its own cabin is $\dfrac{1}{40}$.

If sailor is in his own cabin in the expected value head count will be increased by 1.

$E(X_i) = 1 \times \dfrac{1}{40} + 0 \times \dfrac{39}{40} = \dfrac{1}{40}$; $i = 1, \ldots$ for ith sailor.

$E(X_1 + X_2 + \ldots + X_{40}) = \underbrace{\dfrac{1}{40} + \dfrac{1}{40} + \ldots + \dfrac{1}{40}}_{40 \text{ times}} = 1$

30. (d) $7\cos^2 x + \sin x \cos x - 3 = 0$

$\Rightarrow \quad 7 + \tan x - 3\sec^2 x = 0$

(dividing both sides by $\cos^2 x$)

$\Rightarrow \quad 7 + \tan x - 3(1 + \tan^2 x) = 0$

$\Rightarrow \quad 3\tan^2 x - \tan x - 4 = 0$

$\Rightarrow \quad \tan x = \dfrac{1 \pm \sqrt{1+48}}{6} = \dfrac{1 \pm 7}{6} = -1, \dfrac{4}{3}$

$\Rightarrow \quad \tan x = \tan\dfrac{3\pi}{4}$

or $\quad \tan \tan^{-1}\dfrac{4}{3}$

$\Rightarrow \quad x = n\pi + \dfrac{3\pi}{4}$

or $\quad k\pi + \tan^{-1}\left(\dfrac{4}{3}\right)$

$n, k \in I$

31. (d) By using the known result that the choice of throwing k in a single throw with three dice (or throwing three dice together) is

$$\frac{{}^{k-1}C_2 - 3 \cdot {}^{k-7}C_2}{216}$$

Here, $k = 12$

$\Rightarrow$ Required choice $= \dfrac{{}^{11}C_2 - 3 \cdot {}^{5}C_2}{216} = \dfrac{55-30}{216} = \dfrac{25}{216}$

Alternate method

S. No.	Number of cases of sum of 12	Favourable ways
1.	(1, 5, 6)	$3! = 6$
2.	(2, 6, 4)	$3! = 6$
3.	(2, 5, 5)	$\dfrac{3!}{2!} = 3$
4.	(3, 6, 3)	$\dfrac{3!}{2!} = 3$
5.	(3, 5, 4)	$3! = 6$
6.	(4, 4, 4)	$\dfrac{3!}{3!} = 1$
		Total = 25

$\therefore$ Required probability $= \dfrac{25}{6 \times 6 \times 6} = \dfrac{25}{216}$

32. (b) $I = \int_0^{\frac{\pi}{2}} \frac{dx}{5 + 3\sin x}$

$= \int_0^{\frac{\pi}{2}} \frac{dx}{5\left(\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2}\right) + 3\left(2\sin\frac{x}{2}\cdot\cos\frac{x}{2}\right)}$

$= \int_0^{\frac{\pi}{2}} \frac{\sec^2 \frac{x}{2}\, dx}{5\left(1 + \tan^2 \frac{x}{2} + 6\tan\frac{x}{2}\right)}$

By multiplying both numerator and denominator by $\sec^2 \frac{x}{2}$,

$= \int_0^{\frac{\pi}{2}} \frac{\sec^2 \frac{x}{2}\, dx}{5\left(\tan^2 \frac{x}{2} + \frac{6}{5}\tan\frac{x}{2} + \frac{9}{25}\right) + \frac{16}{5}}$

$= \int_0^{\frac{\pi}{2}} \frac{\sec^2 \frac{x}{2}\, dx}{5\left(\tan\frac{x}{2} + \frac{3}{5}\right)^2 + \frac{16}{5}}$

$= \frac{1}{5}\int_0^{\frac{\pi}{2}} \frac{\sec^2 \frac{x}{2}\, dx}{\left(\tan\frac{x}{2} + \frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2}$

Let $\tan\frac{x}{2} + \frac{3}{5} = y$

$\Rightarrow \frac{1}{2}\sec^2\frac{x}{2}\, dx = dy$

At $x = 0$, $y = \frac{3}{5}$ and at $x = \frac{\pi}{2}$; $y = \frac{8}{5}$

$\therefore \quad I = \frac{1}{5}\int_{\frac{3}{5}}^{\frac{8}{5}} \frac{2dy}{\left(\frac{4}{5}\right)^2 + y^2}$

$= \frac{2}{5}\left[\frac{1}{\left(\frac{4}{5}\right)}\tan^{-1}\left(\frac{y}{\left(\frac{4}{5}\right)}\right)\right]_{\frac{3}{5}}^{\frac{8}{5}}$

$= \frac{1}{2}\left[\tan^{-1} 2 - \tan^{-1}\frac{3}{4}\right]$

$= \frac{1}{2}\left[\tan^{-1}\frac{2 - \frac{3}{4}}{1 + 2\times\frac{3}{4}}\right] = \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow \quad \lambda = \frac{1}{2}$

33. (a) Information which to be send size = 5 bits parity bit would be = 1
Then, total message size = 5 + 1 = 6

34. (a) Probability that any particular coupon will be selected is $\frac{1}{15}$.
Expected number of different types of coupons
$= {}^{15}C_1\left[1 - \left(\frac{14}{15}\right)^5\right] = 15\left[1 - \left(\frac{14}{15}\right)^5\right]$

35. (b) $f(x, y) = \frac{1}{2}ye^{-xy};\ 0 < x < \infty;\ 0 < y < 2$

$E\left[e^{\frac{x}{2}} \middle| y = 1\right] = \int_0^\infty e^{\frac{x}{2}}\cdot\frac{1}{2}e^{-x}\, dx$

$= \int_0^\infty \cdot\frac{1}{2}e^{\frac{-x}{2}}\, dx = \left[-e^{\frac{-x}{2}}\right]_0^\infty = 1$

36. (a) $\sqrt{2\left(\frac{dy}{dx}\right)^3 + 4} = \left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}$

By squaring both sides,

$2\left(\frac{dy}{dx}\right)^3 + 4 = \left(\frac{d^2y}{dx^2}\right)^3$

$\Rightarrow$ Order = 2, Degree = 3

37. (d) Let 'a' be an edge of a cube and 'd' be the diameter of sphere.
Sum of areas of cube and sphere

$A = 6a^2 + \pi d^2 = k$ (constant) ...(i)

Sum of volumes of cube and sphere,

$V = a^3 + \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = a^3 + \frac{\pi d^3}{6}$

$V = a^3\left(1 + \left(\frac{d}{a}\right)^3\cdot\frac{\pi}{6}\right)$ ...(ii)

From Eq. (i),

$6 + \pi\left(\frac{d}{a}\right)^2 = \frac{k}{a^2}$

$\Rightarrow \quad \frac{k}{a^2} = 6 + \pi x^2 \quad \left(\text{let } x = \frac{d}{a}\right)$

$\Rightarrow \quad a^2 = \frac{k}{6 + \pi x^2}$

$\Rightarrow \quad a^3 = \frac{k^{3/2}}{(6 + \pi x^2)^{\frac{3}{2}}}$

$\therefore \quad V = k^{\frac{3}{2}}(6 + \pi x^2)^{\frac{-3}{2}}\left(1 + \frac{\pi}{6}x^3\right)$

V is minimum, if $\frac{dV}{dx} = 0$

$\Rightarrow \quad \frac{dV}{dx} = k^{\frac{3}{2}}\left[(6 + \pi x^2)^{\frac{-5}{2}}\left(\frac{-3}{2}\right)\left(1 + \frac{\pi}{6}x^3\right)(2\pi x) + (6 + \pi x^2)^{\frac{-3}{2}}\left(3x^2\cdot\frac{\pi}{6}\right)\right] = 0$

$\Rightarrow \quad -3\pi x\left(1 + \frac{\pi}{6}x^3\right) + (6 + \pi x^2)\left(\frac{\pi x^2}{2}\right) = 0$

$\Rightarrow \quad 3\left(1 + \frac{\pi}{6}x^3\right) = (6 + \pi x^2)\frac{x}{2}$

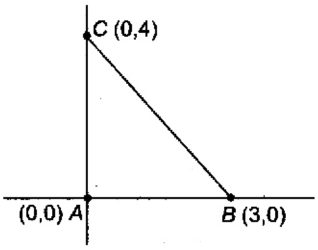
$\Rightarrow \quad 3 + \frac{\pi}{2}x^3 = 3x + \frac{\pi}{2}x^3$

$\Rightarrow \quad x = 1$

$\Rightarrow \quad \frac{d}{a} = 1$

$\Rightarrow \quad d : a = 1 : 1$

38. (a)



Given vertices is right angled triangle in which point of intersection of altitude is (0, 0) which is orthocentre.

39. (c) ROM bit required $= 2^{16} * 10$

or $8 + 8 = 16$ bit to be summed.

40. (a) Circle is $\quad x^2 + y^2 = 2ax \quad \ldots(i)$

$\Rightarrow \quad x^2 + y^2 - 2ax = 0$

$\Rightarrow \quad (x - a)^2 + y^2 = a^2$

Centre of circle is $(a, 0)$ and radius $= a$

A straight line parallel to $x + 2y = 0$ is $x + 2y = c$

It passes through $(a, 0)$, so

$a + 2(0) = c \Rightarrow c = a$

$\therefore \quad x + 2y = a$

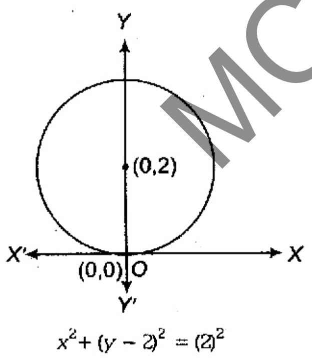
From Eq. (i), $\quad 5y^2 = a^2 \Rightarrow y = \pm \dfrac{a}{\sqrt{5}}$

$\Rightarrow \quad A$ is $\left(a - \dfrac{2a}{\sqrt{5}}, \dfrac{a}{\sqrt{5}}\right)$ and B is $\left(a + \dfrac{2a}{\sqrt{5}}, \dfrac{-a}{\sqrt{5}}\right)$

$\Rightarrow$ Area of ΔAOB; O being $(0, 0)$

$$\therefore \text{Area} = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ a - \dfrac{2a}{\sqrt{5}} & \dfrac{a}{\sqrt{5}} & 1 \\ a + \dfrac{2a}{\sqrt{5}} & -\dfrac{a}{\sqrt{5}} & 1 \end{vmatrix} = \frac{a^2}{\sqrt{5}} \text{ sq units}$$

41. (d) $x^2 + y^2 - 4y = 0$



$\Rightarrow \quad x^2 + (y - 2)^2 = (2)^2$

Centre is (0, 2).

Radius = 2

No portion of circle lies below x-axis, so area = 0 below x-axis.

42. (b)

$ax + hy + gz = 0 \quad \ldots(i)$

$hx + by + fz = 0 \quad \ldots(ii)$

$gx + fy + cz = 0 \quad \ldots(iii)$

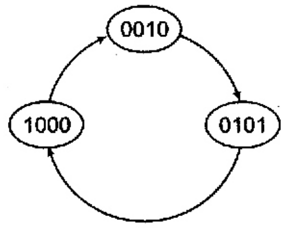
Eliminating x, y, z from the given equations, we get

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

$a(bc - f^2) - h(hc - gf) + g(hf - gb) = 0$

which is being satisfied by option (b) only.

43. (a) In an 8421 counter



but 1100 is an invalid state because it does not come in between the counter sequence.

2 — 5 — 8 — 11

So, next sequence should be (1011).

44. (d) $x = \dfrac{a}{2}\left(\lambda + \dfrac{1}{\lambda}\right); y = \dfrac{b}{2}\left(\lambda - \dfrac{1}{\lambda}\right)$

$\Rightarrow \quad \dfrac{2x}{a} = \left(\lambda + \dfrac{1}{\lambda}\right); \dfrac{2y}{b} = \left(\lambda - \dfrac{1}{\lambda}\right)$

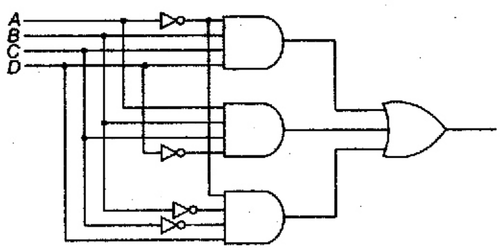
$\Rightarrow \quad \left(\dfrac{2x}{a}\right)^2 - \left(\dfrac{2y}{b}\right)^2 = \left(\lambda + \dfrac{1}{\lambda}\right)^2 - \left(\lambda - \dfrac{1}{\lambda}\right)^2$

$\Rightarrow \quad 4\left(\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2}\right) = 4$

$\Rightarrow \quad \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$; which is a hyperbola.

45. (a) Since, $|F_1 + F_2| \le |F_1| + |F_2|$

46. (c) Expression $\overline{A}BCD + ABC\overline{D} + \overline{ABCD}$.



There are 3 AND gates and 4 inverters.

47. (b) $\dfrac{x^2}{25} + \dfrac{y^2}{b^2} = 1$ is the ellipse, whose foci is $(\pm ae, 0)$,

where $b^2 = a^2(1 - e^2)$

$\Rightarrow \quad e = \sqrt{\dfrac{a^2 - b^2}{a^2}} = \dfrac{1}{a}\sqrt{a^2 - b^2}$

$= (\pm\sqrt{25 - b^2}, 0) \quad (\because a = 5)$

Hyperbola is $\dfrac{x^2}{\left(\dfrac{12}{5}\right)^2} - \dfrac{y^2}{\left(\dfrac{9}{5}\right)^2} = 1$

Foci is $\left(\pm\dfrac{12}{5}e, 0\right); \dfrac{81}{25} = \dfrac{144}{25}(e^2 - 1)$

$\Rightarrow \quad e = \sqrt{\dfrac{81}{144} + 1} = \dfrac{15}{12} = \dfrac{5}{4}$

Foci is $(\pm 3, 0)$.

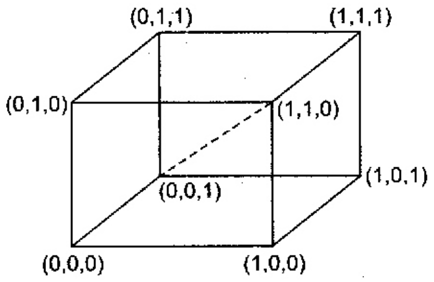
$\Rightarrow \quad \sqrt{25 - b^2} = 3 \Rightarrow b^2 = 16$

48. (b) (*p) + + means the value at address will be incremented, so

a[0] = 1 + 1 = 2

a[1] = 2

49. (c)



Direction cosine of the 4 diagonals are

$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}; -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

and $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$.

Let l, m, n be the direction cosines of the line taken, then

$$\cos\alpha = \frac{1}{\sqrt{3}} l + \frac{1}{\sqrt{3}} m + \frac{1}{\sqrt{3}} n$$

$$\cos\beta = \frac{1}{\sqrt{3}} l - \frac{1}{\sqrt{3}} m + \frac{1}{\sqrt{3}} n$$

$$\cos\gamma = -\frac{1}{\sqrt{3}} l + \frac{1}{\sqrt{3}} m + \frac{1}{\sqrt{3}} n$$

$$\cos\delta = \frac{1}{\sqrt{3}} l + \frac{1}{\sqrt{3}} m - \frac{1}{\sqrt{3}} n$$

$$\therefore \quad \cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$$

$$= \frac{1}{3}[(l+m+n)^2 + (l-m+n)^2 + (-l+m+n)^2 + (l+m-n)^2]$$

$$= \frac{4}{3}[l^2 + m^2 + n^2] = \frac{4}{3}$$

50. (b) When input for J-K flip-flop $J = 1$ and $K = 1$ at 10 kHz clock input, then the output is constantly low.

51. (d) Equation of any tangent to parabola

$y^2 = 8x \Rightarrow y^2 = 4(2)x$

is $y = mx + \frac{2}{m}$ which is tangent to $xy = -1$

$\Rightarrow x\left(mx + \frac{2}{m}\right) + 1 = 0$ has equal roots

$\Rightarrow D = 0 \Rightarrow \frac{4}{m^2} = 4 \cdot m \cdot 1 \Rightarrow m^3 = 1 \Rightarrow m = 1$

(taking real value of m only)

Hence, common tangent is $y = x + 2$.

52. (b) In microprocessor 8085, the temporary registers hold the next address for execution.

53. (b) $x^2 + y^2 - 4x - 2y - 20 = 0 \Rightarrow (x-2)^2 + (y-1)^2 = (5)^2$

So, centre of circle is (2, 1) and radius is 5.

Distance between (2, 1) and (10, 7) is $\sqrt{64+36} = 10$.

Greatest distance = 10 + 5 = 15

54. (a) Given, $\left|\frac{(\mathbf{a} + \lambda\mathbf{b})\cdot\mathbf{c}}{|\mathbf{c}|}\right| = \frac{1}{\sqrt{3}}$

$\Rightarrow \left|\frac{(1+2-1) + \lambda(1-1-1)}{\sqrt{3}}\right| = \frac{1}{\sqrt{3}}$

$\Rightarrow 2 - \lambda = \pm 1 \Rightarrow \lambda = 1$ or $\lambda = 3$

$\Rightarrow$ Required vector is $\mathbf{a} + \mathbf{b}$ or $\mathbf{a} + 3\mathbf{b}$

i.e., $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ or $4\mathbf{i} - \mathbf{j} + 4\mathbf{k}$

55. (d) On a Karnaugh map, grouping the 0's produces a 'don't care' condition.

56. (b) $\mathbf{a} = \frac{2\mathbf{i} + p\mathbf{j} + \mathbf{k}}{\sqrt{5 + p^2}}$ and $\mathbf{b} = \frac{\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}}{\sqrt{14}}$

are orthogonal vectors, if $\mathbf{a}\cdot\mathbf{b} = 0$

$\Rightarrow \quad 2 - 2p + 3 = 0 \Rightarrow p = \frac{5}{2}$

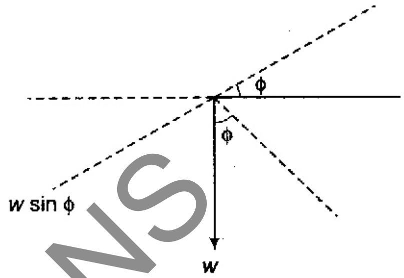
57. (b) A = coefficient of x^n in the expansion of $(1+x)^{2n} = {}^{2n}C_n$

B = coefficient of x^n in the expansion of $(1+x)^{2n-1}$

$= {}^{2n-1}C_n$

$$\frac{A}{B} = \frac{{}^{2n}C_n}{{}^{2n-1}C_n} = \frac{\frac{2n!}{n!n!}}{\frac{(2n-1)!}{n!(n-1)!}} = \frac{2n}{n} = 2$$

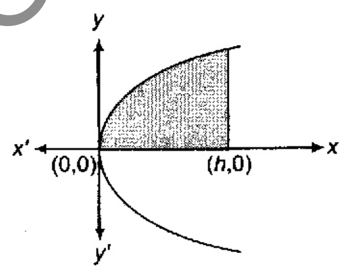
58. (a)



Frictional force = $w \sin\phi$

Which will be nullified for movement.

59. (d)



Let $(\bar{x}, \bar{y})$ be the centre of gravity.

Volume is symmetric about x-axis. So, $\bar{y} = 0$

$$\bar{x} = \frac{\int_0^h x \cdot \pi y^2\, dx}{\int_0^h \pi y^2 dx}$$

$$\Rightarrow \bar{x} = \frac{\int_0^h 4ax^2\, dx}{\int_0^h 4ax\, dx}; \text{ as } y^2 = 4ax$$

$$= \frac{\left[4a \cdot \frac{x^3}{3}\right]_0^h}{\left[4a \cdot \frac{x^2}{2}\right]_0^h} = \frac{2h^3}{3h^2} = \frac{2}{3}h$$

$$\Rightarrow \bar{x} = \frac{2}{3}h; \bar{y} = 0$$

60. (c) With every bit added the magnitude of the number multiplies by 2.

61. (d) Centre of gravity (CG) $= \frac{L}{3} \times \frac{1}{2} = \frac{L}{6}$

Hanging portion weight $= \frac{Mg}{3}$

Now, work done $= \frac{Mg}{3} \times \frac{L}{6} = \frac{MgL}{18}$

62. (d) $I = \int_0^1 \frac{3x}{(x+1)(x-2)} dx$

$$= \int_0^1 \frac{2(x+1) + (x-2)}{(x+1)(x-2)} dx$$

$$= \int_0^1 \left(\frac{2}{x-2} + \frac{1}{x+1}\right) dx$$

$= 2\log(x-2) + \log(x+1)|_0^1$

$= 2\log\left(\frac{1}{2}\right) + \log 2$

$= -\log 2 = \log\frac{1}{2}$

63. (b) It is possible but not permissible for the processes to complete their execution without entering deadlock in any unsafe system.

64. (d) Sequence is $1, -2, 3, -4, 5, -6, \ldots$

nth term of sequence

$u_n = (-1)^{n-1} \cdot n$

$\Rightarrow \lim_{n \to \infty} u_n = +\infty$ or $= -\infty$

$\therefore$ Sequence oscillates infinitely but not convergent.

65. (c) $\forall a \in A$ and $\forall b \in B$

If $a \le b$

As, least upper bound $\le$ greatest lower bound.

66. (c) In a buddy memory management, memory is divided into frame size in some power of 2.

67. (d) Due to uniformly continuous $f(1,5)$ will be an interval and it will contain all the values lying from $f(2)$ to $f(4)$ *i.e.*, contains $[3, 6]$.

68. (c) The number of vertices to which a vertex is associated is known as its degree.

69. (b) Union of any set with null set is that set itself. Hence, union of a Borel set and a null set is a Borel set.

70. (c) $\frac{x}{y} = \frac{3}{4} \Rightarrow y = \frac{4}{3}x$

So, $\frac{7x-4y}{3x+y} = \frac{7x - \frac{16}{3}x}{3x + \frac{4}{3}x} = \frac{\frac{5x}{3}}{\frac{13x}{3}} = \frac{5}{13}$

71. (d) Let $y = ax + \frac{b}{x}$

As, $y = 6$, when $x = 4$

$\Rightarrow 6 = 4a + \frac{b}{4} \Rightarrow 16a + b = 24$ ...(i)

As, $y = \frac{10}{3}$, when $x = 3$

$\frac{10}{3} = 3a + \frac{b}{3} \Rightarrow 9a + b = 10$ ...(ii)

Subtracting Eq. (ii) from Eq. (i), gives

$7a = 14 \Rightarrow a = 2$

$\Rightarrow b = -8 \Rightarrow y = 2x - \frac{8}{x}$

72. (d) $\sqrt{\log_2 x} - 0.5 = \log_2 \sqrt{x}$

$\Rightarrow \sqrt{\log_2 x} - \frac{1}{2} = \frac{1}{2}\log_2 x$ ...(i)

Let $\sqrt{\log_2 x} = y$

So, Eq. (i) becomes

$y - \frac{1}{2} = \frac{1}{2}y^2 \Rightarrow y^2 - 2y + 1 = 0$

$\Rightarrow (y-1)^2 = 0 \Rightarrow y = 1$

$\Rightarrow \log_2 x = 1 \Rightarrow x = 2$

73. (c) Number of pages $= \frac{\text{Memory size}}{\text{Page size}}$

$= \frac{2^{18}}{2^{10}}$

$= 2^8$

$= 256$ pages

74. (d) $(2x + y + 1)\,dx + (4x + 2y - 1)\,dy = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{(2x+y+1)}{(4x+2y-1)} = \frac{-(2x+y+1)}{2(2x+y)-1}$ ...(i)

Put $2x + y = v$

$\Rightarrow 2 + \frac{dy}{dx} = \frac{dv}{dx} \Rightarrow \frac{dy}{dx} = \frac{dv}{dx} - 2$

Putting these values in Eq. (i), we get

$\frac{dv}{dx} - 2 = \frac{-(v+1)}{2v-1}$

$\Rightarrow \frac{dv}{dx} = \frac{-(v+1)}{2v-1} + 2 = \frac{-v-1+4v-2}{2v-1} = \frac{3v-3}{2v-1}$

$\Rightarrow \frac{2v-1}{3v-3}\,dv = dx \Rightarrow \frac{2(v-1)+1}{3(v-1)}\,dv = dx$

$\Rightarrow \int\left(\frac{2}{3} + \frac{1}{3(v-1)}\right)dv = \int dx + C$

$\Rightarrow \frac{2}{3}v + \frac{1}{3}\log(v-1) = x + C$

$\Rightarrow 2v + \log(v-1) = 3x + C$

$\Rightarrow 4x + 2y + \log(2x + y - 1) = 3x + C$

$\Rightarrow \log(2x + y - 1) + x + 2y = C$

75. (c) Let $\tan^{-1}\left(\frac{1}{3}\right) = \theta$

$\Rightarrow \tan\theta = \frac{1}{3}$

$\therefore \sin\left[2\tan^{-1}\left(\frac{1}{3}\right)\right] = \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

$= \frac{\frac{2}{3}}{1+\frac{1}{9}} = \frac{2}{3} \times \frac{9}{10} = \frac{3}{5}$

Again, let $\tan^{-1}(2\sqrt{2}) = \phi$

$\tan\phi = 2\sqrt{2}$

$\therefore \cos[\tan^{-1}(2\sqrt{2})] = \cos\phi$

$= \frac{1}{\sqrt{1+\tan^2\phi}} = \frac{1}{\sqrt{1+8}} = \frac{1}{3}$

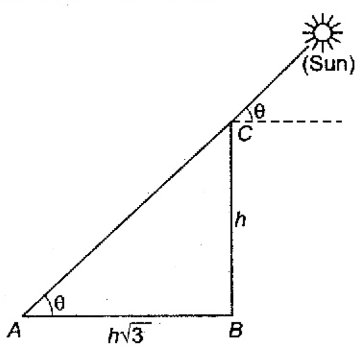
Therefore,

$\sin\left[2\tan^{-1}\left(\frac{1}{3}\right)\right] + \cos[\tan^{-1}(2\sqrt{2})]$

$= \frac{3}{5} + \frac{1}{3} = \frac{9+5}{15} = \frac{14}{15}$

76. (c) By definition of partial order

77. (a) Let h be the height of the pole BC and θ be the angle of elevation of the Sun, then



$\tan\theta = \frac{h}{h\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30°$

78. (c) Let a be the first term and d be the common difference of the AP, then

$$\frac{S_m}{S_n} = \frac{\frac{m}{2}[2a + (m-1)d]}{\frac{n}{2}[2a + (n-1)d]} = \frac{m^2}{n^2}$$

$$\Rightarrow \quad \frac{2a + (m-1)d}{2a + (n-1)d} = \frac{m}{n} \quad \ldots \text{(i)}$$

In Eq. (i) replace m by $2m - 1$ and n by $2n - 1$, we get

$$\frac{2a + (2m-1-1)d}{2a + (2n-1-1)d} = \frac{2m-1}{2n-1}$$

$$\Rightarrow \quad \frac{2[a + (m-1)d]}{2[a + (n-1)d]} = \frac{2m-1}{2n-1}$$

$$\Rightarrow \quad T_m : T_n = 2m - 1 : 2n - 1$$

79. (d) As, $(1-x)^{-r} = 1 + rx + \frac{r(r+1)}{2!}x^2 + \frac{r(r+1)(r+2)}{3!}x^3 + \ldots$

Put $r = 3$, we get

$$(1-x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + \ldots = \frac{1}{(1-x)^3}$$

80. (a) $f(x) = \begin{vmatrix} \sin^2\theta & \cos^2\theta & x \\ \cos^2\theta & x & \sin^2\theta \\ x & \sin^2\theta & \cos^2\theta \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} x+1 & \cos^2\theta & x \\ x+1 & x & \sin^2\theta \\ x+1 & \sin^2\theta & \cos^2\theta \end{vmatrix}$$

$$= (x+1)\begin{vmatrix} 1 & \cos^2\theta & x \\ 1 & x & \sin^2\theta \\ 1 & \sin^2\theta & \cos^2\theta \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$= (x+1)\begin{vmatrix} 1 & \cos^2\theta & x \\ 0 & x - \cos^2\theta & \sin^2\theta - x \\ 0 & \sin^2\theta - \cos^2\theta & \cos^2\theta - x \end{vmatrix}$$

$= (x+1)[-(x - \cos^2\theta)^2 + (x - \sin^2\theta)(\sin^2\theta - \cos^2\theta)]$

$= (x+1)[-x^2 + x\cos^2\theta + x\sin^2\theta - (\cos^4\theta + \sin^4\theta - \sin^2\theta \cdot \cos^2\theta)]$

$= (x+1)[-x^2 + x - (1 - 3\sin^2\theta \cdot \cos^2\theta)]$

$= (x+1)[-x^2 + x - (1 - 3/4)]$, at $\theta = \pi/4$

$= (x+1)\left(-x^2 + x - \frac{1}{4}\right)$

$f(x) = 0 \Rightarrow (x+1)\left(x - \frac{1}{2}\right)^2 = 0$

$\Rightarrow \quad x = -1, \frac{1}{2}, \frac{1}{2}$

81. (b) The character type array string has value "computer science".

c	o	m	p	u	t	e	r		s	c	i	e	n	c	e	\o
ar(0)	ar(1)	ar(2)	ar(3)	ar(4)	5	6	7	8	9	10	11	12	13	14	15	16

*(ar + +) $\Rightarrow$ means or contains the base address and address is incremented the next adjacent address *i.e.*, 0 location to 1 location, so the value at *ar* [1]. It contains O at 1 position.

82. (d) $P(A) = \frac{3}{4}, P(B) = \frac{7}{10}$

$\Rightarrow \quad P(\bar{A}) = \frac{1}{4}, P(\bar{B}) = \frac{3}{10}$

Let $C = A \cap B$

$\Rightarrow \quad P(C) = P(A \cap B) = P(A) \cdot P(B) = \frac{3}{4} \times \frac{7}{10}$

(As, A and B are independent events)

Also, $P(\bar{C}) = P(\bar{A})\, P(\bar{B}) = \frac{1}{4} \times \frac{3}{10}$

$\therefore \quad P\left(\frac{W}{C}\right) = \frac{1}{6}, P\left(\frac{W}{\bar{C}}\right) = \frac{5}{6}$

Now, by using Baye's theorem,

$$P\left(\frac{C}{W}\right) = \frac{P\left(\frac{W}{C}\right)P(C)}{P\left(\frac{W}{C}\right)P(C) + P\left(\frac{W}{\bar{C}}\right)P(\bar{C})}$$

$$= \frac{\frac{1}{6} \times \frac{3}{4} \times \frac{7}{10}}{\frac{1}{6} \times \frac{3}{4} \times \frac{7}{10} + \frac{5}{6} \times \frac{1}{4} \times \frac{3}{10}} = \frac{7}{12}$$

83. (c) $6x^{\frac{3}{4}} = 7x^{\frac{1}{4}} - 2x^{-\frac{1}{4}} \Rightarrow 6x = 7x^{\frac{1}{2}} - 2$

Let $x^{\frac{1}{2}} = y$

$\Rightarrow \quad 6y^2 = 7y - 2$

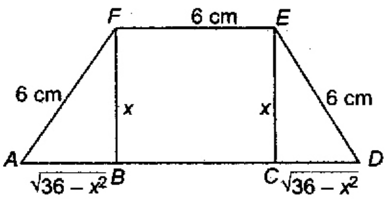
$\Rightarrow \quad 6y^2 - 7y + 2 = 0$

$\Rightarrow \quad (2y-1)(3y-2) = 0$

$\Rightarrow \quad y = \frac{1}{2}, \frac{2}{3} \quad \Rightarrow x^{\frac{1}{2}} = \frac{1}{2}, \frac{2}{3}$

$\Rightarrow \quad x = \frac{1}{4}, \frac{4}{9}$

84. (a)



Area of trapezium $= \frac{1}{2}$ (sum of parallel sides)

$\times$ (perpendicular distance between them)

$= \frac{1}{2}(6 + 6 + 2\sqrt{36 - x^2})\, x$

$= (6 + \sqrt{36 - x^2})x$

$\Rightarrow \quad A = 6x + x\sqrt{36 - x^2}$

For maxima, $\frac{dA}{dx} = 0$

$\frac{dA}{dx} = 0 \quad \Rightarrow 6 + \sqrt{36 - x^2} + \frac{x(-2x)}{2\sqrt{36 - x^2}} = 0$

$\Rightarrow \quad 6 + \frac{36 - 2x^2}{\sqrt{36 - x^2}} = 0$

$\Rightarrow \quad 6\sqrt{36 - x^2} = -2(18 - x^2)$

$\Rightarrow \quad 9(36 - x^2) = (18 - x^2)^2$

$\Rightarrow \quad x^4 - 27x^2 = 0$

$\Rightarrow \quad x^2(x^2 - 27) = 0$

$\Rightarrow \quad x = 3\sqrt{3}$

$\Rightarrow \quad A_{max} = 6(3\sqrt{3}) + 3\sqrt{3}\sqrt{36 - 27} = 27\sqrt{3}$

85. (d) Size of inverted page table

$$= \frac{\text{Size of physical memory}}{\text{Page size}} = \frac{2^{32}}{8\text{ K}}$$

$= \dfrac{2^{32}}{2^3 \times 2^{10}}$

$= \dfrac{2^{32}}{2^{13}} = 2^{19}$ pages

86. (c) $\sin(x + 3\alpha) = 3\sin(\alpha - x)$

$\Rightarrow \sin x \cos 3\alpha + \cos x \sin 3\alpha = 3(\sin\alpha \cos x - \cos\alpha \sin x)$

$\Rightarrow \sin x(\cos 3\alpha + 3\cos\alpha) = \cos x(3\sin\alpha - \sin 3\alpha)$

$\Rightarrow \tan x = \dfrac{3\sin\alpha - \sin 3\alpha}{\cos 3\alpha + 3\cos\alpha} = \dfrac{4\sin^3\alpha}{4\cos^3\alpha}$

$\Rightarrow \tan x = \tan^3\alpha$

87. (c) The rule of inference

$a \to b$
$\underline{b \to c}$
$a \to c$

is known as syllogism.

88. (d) $y(x^2y + e^x)\,dx - e^x dy = 0$

Dividing both sides by y^2, we get

$$\left(x^2 + \frac{e^x}{y}\right)dx - \frac{e^x}{y^2}dy = 0$$

$$\Rightarrow \quad x^2dx + \frac{ye^x dx - e^x dy}{y^2} = 0$$

$$\Rightarrow \quad d\left(\frac{x^3}{3}\right) + d\left(\frac{e^x}{y}\right) = 0$$

$$\Rightarrow \quad \int d\left(\frac{x^3}{3}\right) + \int d\left(\frac{e^x}{y}\right) = C$$

$$\Rightarrow \quad \frac{x^3}{3} + \frac{e^x}{y} = C \quad \Rightarrow \quad x^3y + 3e^x = Cy$$

89. (c) Given function,

$$f(x) = x\left[1 + \frac{1}{3\sin(\log x^2)}\right], x \neq 0$$

and given $f(0) = 0$

LHL $= f(0 - 0) = \lim_{h \to 0} f(0 - h)$

$$= \lim_{h \to 0} (-h)\left[1 + \frac{1}{3\sin(\log h^2)}\right] \quad (\because -1 \le \sin\theta \le 1)$$

$$= \lim_{h \to 0} -h\left(1 + \frac{1}{6\sin(\log h)}\right)$$

$= -0 \times$ (finite value) $= 0$

RHL $= f(0 + 0) = \lim_{h \to 0} f(0 + h)$

$$= \lim_{h \to 0} h\left[1 + \frac{1}{3\sin(\log h^2)}\right]$$

$= 0 \times$ (finite value) $= 0$

So, $f(x)$ is continuous at $x = 0$.

Now, $Rf'(0) = \lim_{h \to 0} \dfrac{f(0 + h) - f(0)}{h}$

$$= \lim_{h \to 0} \frac{h\left\{1 + \frac{1}{3\sin(\log h^2)}\right\} - 0}{h}$$

= (finite value lie between some value)

= does not exist

Similarly, $Lf'(0)$ = does not exist.

So, $f(x)$ is not differentiable at $x = 0$.

90. (d) $h(x) = [f(x)]^2 + [g(x)]^2$

$\Rightarrow \quad h'(x) = 2f(x)\,f'(x) + 2g(x)\,g'(x)$

$= -2f'(x)\,f'(x) + 2f'(x)\,f''(x)$

[Given, $f''(x) = -f(x)$, $f'(x) = g(x)$]

$\Rightarrow \quad h'(x) = 0$

$\Rightarrow \quad h(x) = $ constant

$\Rightarrow \quad h(10) = h(5) = 11$

91. (b) $\cot A - \tan A = \cot A - \dfrac{1}{\cot A}$

$= \dfrac{\cot^2 A - 1}{\cot A} = \dfrac{\cos^2 A - \sin^2 A}{\cos A \sin A} = 2\cot 2A$

$\Rightarrow \tan A = \cot A - 2\cot 2A$

Now, $\tan A + 2\tan 2A + 4\tan 4A + 8\cot 8A$

$= (\cot A - 2\cot 2A) + 2(\cot 2A - 2\cot 4A) + 4(\cot 4A - 2\cot 8A) + 8\cot 8A$

$= \cot A$

92. (d) In isolated mapped IO scheme, the pin $M/\overline{IO}$ helps in isolating memory with the IO devices and also in increasing the number of addressable memory locations and also in increasing the number of I/O devices.

93. (c) The required probability

$$= {}^4C_2 \times \left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 + {}^4C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right) + {}^4C_4\left(\frac{1}{6}\right)^4$$

$$= \frac{4 \times 3}{2} \times \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} + 4 \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} + \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{25}{216} + \frac{5}{324} + \frac{1}{1296} = \frac{150 + 20 + 1}{1296} = \frac{171}{1296} = \frac{19}{144}$$

94. (c) $y = \sin^3\theta$

$$\Rightarrow \quad \frac{dy}{d\theta} = 3\sin^2\theta\cos\theta$$

$$x = \cos\theta \Rightarrow \frac{dx}{d\theta} = -\sin\theta$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = -3\sin\theta\cos\theta$$

So, $$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{d\theta}\left(\frac{dy}{dx}\right)}{\frac{dx}{d\theta}}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-3\cos^2\theta + 3\sin^2\theta}{-\sin\theta}$$

Now, $y\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2$

$= -\sin^2\theta(-3\cos^2\theta + 3\sin^2\theta) + 9\sin^2\theta\cos^2\theta$

$= 3\sin^2\theta\cos^2\theta - 3\sin^4\theta + 9\sin^2\theta\cos^2\theta$

$= 3\sin^2\theta[\cos^2\theta - \sin^2\theta + 3\cos^2\theta]$

$= 3\sin^2\theta[5\cos^2\theta - 1]$

95. (b) When $n = 1000$ (size of file)

then, $\log n$ = time taken to search an item

Now, $n = 1000000000$

time $= \log 1000000000 = 9$ ms

96. (c) 15 K entries are required in FAT.

97. (b) $I = \displaystyle\int \frac{x}{\sec x + 1}dx = \int \frac{x\cos x}{1 + \cos x}dx$

$$= \int \frac{x[(1 + \cos x) - 1]}{(1 + \cos x)}dx = \int \left(x - \frac{x}{1 + \cos x}\right)dx$$

$$= \frac{x^2}{2} - \int \frac{x}{2\cos^2\frac{x}{2}}dx = \frac{x^2}{2} - \frac{1}{2}\int x\sec^2\frac{x}{2}\,dx$$

$$= \frac{x^2}{2} - \frac{1}{2}\left[x \cdot 2\tan\frac{x}{2} - \int 2\tan\frac{x}{2}\,dx\right] \quad \text{(by parts)}$$

$$= \frac{x^2}{2} - x\tan\left(\frac{x}{2}\right) + \int \tan\frac{x}{2}\,dx$$

$$= \frac{x^2}{2} - x\tan\left(\frac{x}{2}\right) + 2\log\left(\sec\frac{x}{2}\right)$$

98. (a) An entity and its subclasses and their subclasses and so on is called as type hierarchy.

99. (c) $I = \int \frac{x - \sin x}{1 - \cos x}\,dx$

$$= \int \frac{x}{1-\cos x}\,dx - \int \frac{\sin x}{1 - \cos x}\,dx$$

$$= \int \frac{x}{2\sin^2\frac{x}{2}}\,dx - \int \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\,dx$$

$$= \frac{1}{2}\int x\,\text{cosec}^2\frac{x}{2}\,dx - \int \cot\frac{x}{2}\,dx = I_1 - I_2$$

$$I_1 = \frac{1}{2}\left[x\int \text{cosec}^2\frac{x}{2}\,dx - \int\left(1 \times \int \text{cosec}^2\frac{x}{2}\,dx\right)dx\right]$$

$$= \frac{1}{2}\left[x\left(-2\cot\frac{x}{2}\right) + \int 2\cot\frac{x}{2}\,dx\right]$$

$$= -x\cot\frac{x}{2} + I_2$$

$$\Rightarrow I_1 - I_2 = -\cot\frac{x}{2} \quad \therefore \quad I = -x\cot\frac{x}{2}$$

100. (d) For $0 < A < 1$

$$\lim_{x \to \infty} A^x = 0$$

$$\Rightarrow \lim_{x \to \infty} A^x \sin\left(\frac{B}{A^x}\right)$$

$= 0 \times$ (finite value between -1 and $+1$)

$= 0$

101. (d) Given differential equation is

$$\frac{dy}{dx} = \frac{3x - 4y - 2}{3x - 4y - 3} \quad \ldots\text{(i)}$$

Put, $3x - 4y = v$

$$\Rightarrow 3 - 4\frac{dy}{dx} = \frac{dv}{dx}$$

Now, differential Eq. (i) becomes

$$\frac{1}{4}\left(3 - \frac{dv}{dx}\right) = \frac{v-2}{v-3}$$

$$\Rightarrow \frac{dv}{dx} = 3 - \frac{4v-8}{v-3} = \frac{3v - 9 - 4v + 8}{v-3}$$

$$= \frac{-v-1}{v-3} = \frac{v+1}{3-v}$$

$$\Rightarrow \frac{3-v}{v+1}\,dv = dx$$

$$\Rightarrow \left(\frac{3}{v+1} - \frac{v+1-1}{v+1}\right)dv = dx$$

$$\Rightarrow \left(\frac{4}{v+1} - 1\right)dv = dx$$

$$\Rightarrow 4\int\frac{dv}{v+1} - \int dv = \int dx$$

$$\Rightarrow 4\log(v+1) - v = x + C$$

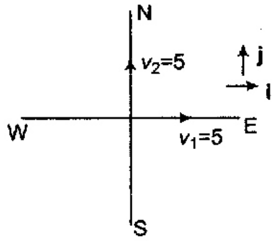
$$\Rightarrow 4\log(3x - 4y + 1) - (3x - 4y) = x + C$$

$$\Rightarrow 4\log(3x - 4y + 1) = 4x - 4y + 4C' \quad (\text{put } C = 4C')$$

$$\Rightarrow \log(3x - 4y + 1) = x - y + C$$

102. (d) $a = +1$ is not the correct C statement.

103. (b) Acceleration $= \frac{\Delta v}{\Delta t}$



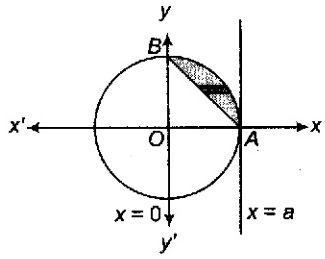
$$= \frac{v_2 - v_1}{t} = \frac{5\mathrm{j} - 5\mathrm{i}}{10} = -\frac{1}{2}\mathrm{i} + \frac{1}{2}\mathrm{j}$$

In the North-West direction with magnitude

$$= \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{1}{4}} = \frac{1}{\sqrt{2}}\ \text{m/s}^2$$

104. (a) Domain integrity constraint guarantees that every primary key attribute is non-null.

105. (b) Shaded portion is the required region bounded by the curves $x^2 + y^2 = a^2$ and $x + y = a$ in the first quadrant which is given by



$$\int_{x=0}^{x=a}\left(\int_{y=a-x}^{y=\sqrt{a^2-x^2}} dy\right)dx$$

$$= \int_0^a \int_{a-x}^{\sqrt{a^2-x^2}} dy\,dx$$

106. (c) Integrity rule identifies the cell.

107. (c) $A = \begin{bmatrix} 5 & 0 & -2 \\ 0 & 1 & 0 \\ -4 & 0 & -1 \end{bmatrix}$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$I - A = \begin{bmatrix} -4 & 0 & 2 \\ 0 & 0 & 0 \\ 4 & 0 & 2 \end{bmatrix}$$

$R_3 \to R_3 + R_1$

$$= \begin{bmatrix} -4 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

which has two non-zero rows.

$\Rightarrow$ Rank of $I - A = 2$

108. (a) Loader combines the output of compiler with various library functions to produce an executable image.

109. (d) $AB \neq BA$ in general.

110. (b) $\Delta = \begin{vmatrix} \tan A & 1 & 1 \\ 1 & \tan B & 1 \\ 1 & 1 & \tan C \end{vmatrix}$

$= \tan A (\tan B \tan C - 1) - 1 (\tan C - 1) + 1 (1 - \tan B)$
$= \tan A \tan B \tan C - \tan A - \tan B - \tan C + 2$
$= 2$
(since, in ΔABC, $\tan A + \tan B + \tan C$
$= \tan A \tan B \tan C$)

111. (d) Since, x having precedence over +operation

Expression

$((2 + 3) \times 4 + 5 \times (6 + 7) \times 8) + 9$
$(+ 23 \times 4 + 5 + 67 \times 8) + 9$
$(\overline{\times + 234} + 5 \overline{\times \times + 678}) + 9$
$(\overline{\times + 234} + \overline{\times 5 \times + 678}) + 9$
$\overline{+ \times + 234 \times 5 \times + 678} + 9$
$\overline{+ + \times + 234 \times 5 \times + 6789}$

112. (d) $z = \begin{vmatrix} 1 & e^{\frac{i\pi}{4}} & e^{\frac{i\pi}{3}} \\ e^{-\frac{i\pi}{4}} & 1 & e^{\frac{i2\pi}{3}} \\ e^{-\frac{i\pi}{3}} & e^{-\frac{i2\pi}{3}} & 1 \end{vmatrix}$ $(\because e^{-2\pi i} = 1)$

$\Rightarrow \quad \bar{z} = \begin{vmatrix} 1 & e^{-\frac{i\pi}{4}} & e^{-\frac{i\pi}{3}} \\ e^{\frac{i\pi}{4}} & 1 & e^{-\frac{i2\pi}{3}} \\ e^{\frac{i\pi}{3}} & e^{\frac{i2\pi}{3}} & 1 \end{vmatrix}$

Changing all rows into columns

$= \begin{vmatrix} 1 & e^{\frac{i\pi}{4}} & e^{\frac{i\pi}{3}} \\ e^{-\frac{i\pi}{4}} & 1 & e^{\frac{i2\pi}{3}} \\ e^{-\frac{i\pi}{3}} & e^{-\frac{i2\pi}{3}} & 1 \end{vmatrix} = z$

$\Rightarrow \bar{z} = z$
$\Rightarrow z$ is purely real.
$\Rightarrow \quad \text{Im}(z) = 0 \Rightarrow y = 0$

113. (d) $\begin{vmatrix} 1 & 1 + i & i \\ 1 + i & i & 1 \\ i & 1 & 1 + i \end{vmatrix}$

$= 1 (i + i^2 - 1) - (1 + i) [(1 + i)^2 - i] + i (1 + i - i^2)$
$= (i - 2) - (i + 1) (i) + i (2 + i)$
$= i - 2 - i + 1 + 2i + i^2 = -2 + 2i$

114. (d) Given an n-bit number, we can represent two's complement numbers in the range

$$-2^{n-1} \text{ to } 2^{n-1} - 1$$

115. (b) Given differential equation is

$$y^5 x + y - x \frac{dy}{dx} = 0$$

$\Rightarrow \quad x \frac{dy}{dx} = y^5 x + y$

$\Rightarrow \quad \frac{dy}{dx} = y^5 + \frac{1}{x} \cdot y$

$\Rightarrow \quad \frac{dy}{dx} - \frac{1}{x} y = y^5 \Rightarrow y^{-5} \frac{dy}{dx} - \frac{1}{x} y^{-4} = 1$

Put $\quad y^{-4} = z$

$\Rightarrow \quad -4 y^{-5} \frac{dy}{dx} = \frac{dz}{dx}$

$\therefore \quad -\frac{1}{4} \frac{dz}{dx} - \frac{1}{x} \cdot z = 1$

$\Rightarrow \quad \frac{dz}{dx} + \frac{4}{x} z = -4$

which is a linear differential equation.

$$\text{IF} = e^{\int \frac{4}{x} dx} = e^{4 \log x}$$
$$= e^{\log x^4} = x^4$$

$\therefore$ Solution is

$$z \cdot x^4 = -4 \int x^4 \, dx$$

$\Rightarrow \quad zx^4 = -4 \frac{x^5}{5} + C$

$\Rightarrow \quad \left(\frac{x}{y}\right)^4 = -4 \frac{x^5}{5} + C \Rightarrow \frac{x^5}{5} + \frac{1}{4} \left(\frac{x}{y}\right)^4 = C$

116. (c) Correlated subquery is resolved in the top to bottom fashion.

117. (b) $\sqrt{x^2 + \sqrt{x^2 + 11}} + \sqrt{x^2 - \sqrt{x^2 + 11}} = 4$ ...(i)

$\Rightarrow \quad x^2 + \sqrt{x^2 + 11} + x^2 - \sqrt{x^2 + 11}$
$+ 2 \sqrt{x^4 - (x^2 + 11)} = 16$

By squaring both sides of Eq. (i),

$\Rightarrow \quad x^2 + \sqrt{x^4 - x^2 - 11} = 8$
$\Rightarrow \quad x^2 - 8 = \sqrt{x^4 - x^2 - 11}$
$\Rightarrow \quad (x^2 - 8)^2 = x^4 - x^2 - 11$
$\Rightarrow \quad x^4 - 16x^2 + 64 = x^4 - x^2 - 11$
$\Rightarrow \quad 15x^2 = 75$
$\Rightarrow \quad x^2 = 5$
$\Rightarrow \quad x = \pm \sqrt{5}$

i.e., two irrational solutions.

118. (b) $(r + 1)$th term in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{12}$ is

$$T_{r+1} = {}^{12}C_r (2x^2)^{12-r} \left(-\frac{1}{x}\right)$$
$$= {}^{12}C_r \, 2^{12-r} \cdot x^{12-3r} (-1)^r$$

For term independent of x

$12 - 3r = 0 \Rightarrow r = 4$

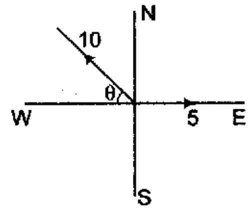
i.e., $T_5 = (-1)^4 \, 2^4 \cdot {}^{12}C_4 = 16 \times \frac{12 \times 11 \times 10 \times 9}{24} = 7920$

119. (c) Required probability

$= \frac{1}{6} + \left(\frac{5}{6}\right)^2 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \ldots = \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \ldots\right]$

$= \frac{1}{6} \times \frac{1}{1 - \left(\frac{5}{6}\right)^2} = \frac{1}{6} \times \frac{36}{11} = \frac{6}{11}$

120. (c) From figure



$\cos \theta = \frac{1}{2} \Rightarrow \theta = 60°$

Man should win $(90° - 60°) = 30°$ West of North.