JNU MCA
Solved Paper 2008

1. Let $\{X_n\}$ and $\{Y_n\}$ denote two sequences of integers defined as follows
$X_0 = 1,\ X_1 = 1,\ X_{n+1} = X_n + 2X_{n-1};\ n = 1, 2, \ldots$
$Y_0 = 1,\ Y_1 = 7,\ Y_{n+1} = 2Y_n + 3Y_{n-1};\ n = 1, 2, \ldots\ldots$
How many terms are there which occur in both sequences?
(a) 1 (b) 3
(c) 8 (d) None of these

2. Let I_A and I_B be indicator variables for the events A and B such that

$$I_A = \begin{cases} 1, & \text{if } A \text{ occurs} \\ 0, & \text{otherwise} \end{cases} \quad I_B = \begin{cases} 1, & \text{if } B \text{ occurs} \\ 0, & \text{otherwise} \end{cases}$$

The covariance of I_A and I_B is
(a) $P(AB)$
(b) $P(AB) - P(A)\,P(B)$
(c) $P(A)\,P(B)$
(d) $1 - P(A) - P(B)$

3. $\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} |x - \mu| \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] dx$ equals to
(a) σ (b) 0
(c) $(2/\pi)^{1/2}\sigma$ (d) $\mu + \sigma$

4. Two sides of a triangle are formed by the vectors $\mathbf{a} = 3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - \mathbf{j} + 3\mathbf{k}$. One of the angle of the triangle is given by
(a) $\cos^{-1}\frac{7}{\sqrt{75}}$ (b) $\cos^{-1}\frac{3}{\sqrt{15}}$
(c) $\cos^{-1}\frac{2}{3}$ (d) None of these

5. $\int \frac{\log x^2}{x} dx$ is equal to
(a) $\frac{(\log x)^2}{2} + C$ (b) $\frac{(\log x)^2}{3} + C$
(c) $\frac{(\log x)^2}{4} + C$ (d) $\frac{(\log x^2)^2}{4} + C$

6. If X_1 and X_2 are independent binomial variates with parameters $n_1 = 3,\ p_1 = 1/3$ and $n_2 = 5,\ p_2 = 1/3$, then $P(X_1 + X_2 \geq 1)$ is
(a) 1/16 (b) $(2/3)^8$
(c) $1 - (2/3)^8$ (d) 1/32

7. Structure is a programming language concept for aggregation of data using Cartesian product through conjuction of fields in most programming languages. Which one of the following is used for aggregation of data using Cartesian product through disjunction of its field?
(a) Array (b) Pointer
(c) String (d) Union

8. If $A > 0$, $B > 0$ and $A + B = \pi/3$, then the maximum value of $\tan A \tan B$ is
(a) 0 (b) 1/3
(c) 3 (d) None of these

9. $\int x \tan^{-1} x dx$ is equal to
(a) $\frac{(x^2+1)\tan^{-1}x}{2} - x + C$
(b) $\frac{(x^2+1)\tan^{-1}x - x}{2} + C$
(c) $\frac{-(x^2+1)\tan^{-1}x + x}{2} + C$
(d) None of the above

10. If $\mathbf{d} = \lambda(\mathbf{a}\times\mathbf{b}) + \mu(\mathbf{b}\times\mathbf{c}) + \eta(\mathbf{c}\times\mathbf{a})$ and $[\mathbf{a}, \mathbf{b}, \mathbf{c}] = 1/8$, then $\lambda + \mu + \eta$ is equal to
(a) $(a + b + c)$ (b) $(a \cdot b \times c)$
(c) $(a \times b \times c)$ (d) None of these

11. The volume of the tetrahedron whose vertices are the points with position vectors $\mathbf{i} - 6\mathbf{j} + 10\mathbf{k}$, $-\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$, $5\mathbf{i} - \mathbf{j} + \lambda\mathbf{k}$ and $7\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}$ is 11 cu units, if the value of λ is
(a) -1 (b) 1
(c) -7 (d) None of these

12. If $z = x + iy$, $z^{1/3} = a - ib$, $a \neq \pm ab$, $b \neq 0$, then $bx - ay = kab(a^2 - b^2)$ where k is equal to
(a) 1 (b) 2 (c) 3 (d) 4

13. The value of $\sum_{k=1}^{10}\left(\sin\frac{2\pi k}{11} - i\cos\frac{2\pi k}{11}\right)$ is
(a) 1 (b) 0 (c) $-i$ (d) i

14. The value of a for which the quadratic equation $3x^2 + 2(1 + a^2)x + (a^2 - 3a + 2) = 0$ possesses roots of opposite sign lies in
(a) $(-\infty, 1)$ (b) $(-\infty, 0)$
(c) $(1, 2)$ (d) $(1.5, 2)$

15. The equation $\cos 2x + a\sin x = 2a - 7$ possesses a solution, if
(a) $a < 2$ (b) $2 \leq a \leq 6$
(c) $a > 6$ (d) a is any integer

16. For $0 < a < \pi/2$, if $x = \sum_{n=0}^{\infty} \cos^{2n} a, y = \sum_{n=0}^{\infty} \sin^{2n} a$, $z = \sum_{n=0}^{\infty} \cos^{2n} a \sin^{2n} a$, then

(a) $xyz = xz + y$ (b) $x + y + z + xyz = 0$
(c) $xyz = xy + z$ (d) $xy^2 + x^2y = z$

17. Two rays are drawn through a point at an angle of 30°. A point B is taken on one of the them at a distance d from the point A. A perpendicular is drawn from the point B to the other ray and another perpendicular is drawn from as foot to meet AB at another point from where the similar process is repeated indefinitely. The length of the resulting infinite polygonal line is equal to

(a) $d(2-\sqrt{3})$ (b) $d(2+\sqrt{3})$
(c) infinite (d) None of these

18. The expression

$\cos^2(A-B) + \cos^2 B - 2\cos(A-B)\cos A \cos B$ is

(a) dependent of A
(b) dependent of B
(c) dependent of A and B
(d) None of the above

19. If x is the value of $\tan 3A \cot A$, then

(a) $x < 1$ (b) $1/3 < x < 3$
(c) $0 < x < 1$ (d) None of these

20. If $\tan A = 5/6$ and $\tan B = 1/11$, then

(a) $A + B = \pi/6$ (b) $A + B = \pi/4$
(c) $A + B = \pi/3$ (d) None of these

21. Choose one number which is similar to the numbers in the given set

4718, 5617, 6312, 8314

(a) 2715 (b) 3410
(c) 5412 (d) 6210

22. The hexadecimal of 756.603 with base 8 is

(a) IEE.C18 (b) 2F4.25B
(c) 3DD.83 (d) None of these

23. In a triangle, the lengths of the two larger sides are 10 and 9 respectively. If the angles are in AP, the length of the third side can be

(a) $3\sqrt{5}$ (b) $5\sqrt{3}$
(c) $5+\sqrt{6}$ (d) None of these

24. If two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ cut the coordinate axes in concyclic points, then

(a) $a_1a_2 + b_1b_2 = 0$ (b) $a_1a_2 - b_1b_2 = 0$
(c) $a_1b_1 + a_2b_2 = 0$ (d) $a_1b_1 - a_2b_2 = 0$

25. Consider the following statements

I. Static languages do not support recursion.
II. The memory requirement for stack-based language such as ALGOL-60 can be estimated at compile time.
III. Resolution of overloaded operators can be done at translation time.

Which one of the following options is correct?

(a) I and II are true
(b) I and III are true
(c) II and III are true
(d) I, II and III are true

26. What will the following program do?

```
# include <stdio.h>
main ( )
{
char *names[6];
int i;
for (i = 0, i<=5; i++)
{
printf("\n Enter name");
scanf("%s", names[i]);
}
}
```

(a) The program does not work properly.
(b) The program gives syntax error.
(c) The program reads 6 strings.
(d) None of the above

27. For a memory chip having capacity of 32 kilobytes, the minimum number of address lines required is

(a) 5 (b) 10 (c) 15 (d) 32

28. Determine number from the given alternatives, having the same relation with this number as the numbers of the given pair bear in the given 7528 : 5362 : : 4673 : ?

(a) 2367 (b) 2451
(c) 2531 (d) None of these

29. How many terms are there in the series?

201, 208, 215,......, 369

(a) 23 (b) 24 (c) 25 (d) 26

30. Let R (ABCDEH) and F = {A → BC, CD → E, E → C, AH → D}. Which of the following is not correct?

(a) A and H are prime
(b) B, C, D and E are non-prime
(c) AH is only candidate key
(d) DE is only candidate key

31. A man said to a lady, "Your mother's husband's sister is my aunt." How is the lady related to the man?

(a) Daughter (b) Granddaughter
(c) Mother (d) Sister

32. The solution of $y = x\left(\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3\right)$ is

(a) $ye^{-\frac{1}{2p^2}} = (1+p^{-2})$ (b) $y = p^{-3}e^{\frac{1}{2p^2}}(p+p^3)$
(c) $y = p^3e^{-\frac{1}{2p^2}}(p+p^3)$ (d) $ye^{-\frac{1}{2p^2}} = (1+p^{-2})$

33. Let $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ and $\mathbf{c} = c_1\mathbf{i} + c_2\mathbf{j} + c_3\mathbf{k}$ be three non-zero vectors such that $\mathbf{c}$ is a unit vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$. If the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\pi/6$, then

$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to

(a) 0
(b) 1
(c) $\frac{1}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$
(d) $\frac{3}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)(c_1^2 + c_2^2 + c_3^2)$

34. Determine which of the following is not true.

I. The rate of convergence of Regula-Falsi method is 1.

II. The secant method, when it converges it does so with approximate rate of convergence 1.62.

III. Regula-Falsi method always converges.

IV. Assume that initial guess is very close to the root, Newton-Raphson method always converges when applied to $f(x) = 0$.

(a) I (b) II
(c) III (d) IV

35. Find all real numbers t for which the quadratic form Q defined by $Q(x_1, x_2, x_3) = 2x_1^2 + x_2^2 + 3x_3^2 + 2tx_1x_2 + 2x_1x_3$ is positive definite.

(a) $t > 1$ (b) $t < 0$
(c) $|t| < (5/2)^{1/2}$ (d) $|t| < (5/3)^{1/2}$

36. Let $f(1) = 1$ and $f(n) = 2\sum_{r=1}^{n-1} f(r)$. Then, $\sum_{n=1}^{m} f(n)$ is equal to

(a) $3^{m-1} - 1$ (b) 3^{m-1}
(c) $3m - 1$ (d) None of these

37. $\lim_{x \to 1} \frac{\sum_{r=1}^{n} x^r - n}{x - 1}$ is equal to

(a) $n/2$ (b) $n(n+1)/2$
(c) 1 (d) None of these

38. $\int_0^{2\pi} \sin x \cos 2x \, dx$ is equal to

(a) 1 (b) 2
(c) 4 (d) 0

39. The area included between the parabola $y^2 = 4ax$ and $x^2 = 4ay$ is equal to

(a) $8a^2/3$ (b) $16a^2/3$
(c) $4a^2/3$ (d) None of these

40. Let a, b and c be three non-zero vectors such that $a + b + c = 0$ and $|a| = 3, |b| = 5$ and $|c| = 7$. Then, an angle between a and b is

(a) 15° (b) 30°
(c) 45° (d) 60°

41. If $x_r = \cos\left(\frac{\pi}{2^r}\right) + i \sin\left(\frac{\pi}{2^r}\right)$, then $x_1 x_2 x_3 \ldots \infty$ is equal to

(a) −3 (b) −2
(c) −1 (d) 0

42. If GIVE is coded as 5137 and BAT is coded as 924, how is GATE coded?

(a) 5427 (b) 5724
(c) 5247 (d) 2547

43. The angle of elevation of a cloud from a point x metre above a lake is A and the angle of depression of its reflection in the lake is 45°. The height of the cloud is

(a) $x \tan(A)$
(b) $x \tan(45°)$
(c) $x \tan(A + 45°)$
(d) $x \cot(A + 45°)$

44. The coordinates of A, B, C and D are (6, 3), (−3, 5), (4, −2) and $(x, 3x)$ respectively. If the area of the ΔABC is twice that of the ΔDBC, the value of x can be

(a) −3/8 (b) −11/2
(c) 11/8 (d) 4

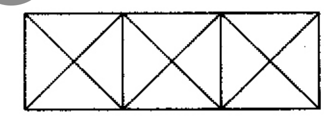
45. The equation of a tangent to the parabola $y^2 = 8x$ which makes an angle 45° with the line $y = 3x + 5$, is

(a) $2x + y + 1 = 0$ (b) $y = 2x + 1$
(c) $x + 2y - 8 = 0$ (d) None of these

46. Bob writes down a number between 1 and 1000. Mary must identify that number by asking 'yes/no' questions of Bob. Mary knows that Bob always tells the truth. If Mary uses an optimal strategy, then how many questions she must ask to determine the answer at the end in the worst case?

(a) 1000 (b) 999
(c) 10 (d) 32

47. How many triangles and squares are there in the following figure?



(a) 28 triangles, 5 squares
(b) 24 triangles, 4 squares
(c) 28 triangles, 4 squares
(d) 24 triangles, 5 squares

48. AFHO : GBDM :: CHFM : ?

(a) GBLD (b) GBJO
(c) GPLD (d) GBDL

49. Which of the following is the fastest IPC mechanism?

(a) FIFO (b) Pipes
(c) Semaphore (d) Mailboxes

50. Which one of the following is not used to define the syntax rules of a programming language?

(a) Binary normal form
(b) Backus-Naur form
(c) EBNF
(d) Syntax diagram

51. Aamir walks 10 km towards North. From there he walks 6 km towards South. Then, he walks 3 km towards East. How far and in which direction is he with reference to his starting point?

(a) 5 km North-West
(b) 7 km West
(c) 7 km East
(d) 5 km North-East

52. In a row of 21 boys, when Raj was shifted by four places towards the right, he became 12th from the left end. What was his earlier position from the right end of the row?

(a) 11th (b) 12th
(c) 13th (d) 14th

53. Which of the following statements is/are correct?

(a) A heap is always a complete binary tree.
(b) An AVL tree is always a binary search tree.
(c) Full binary tree is a special case of complete binary.
(d) All of the above

54. The 8-bit 2's complement of 45 is
(a) 00101101
(b) 11010010
(c) 11010011
(d) 10101101

55. The instruction JNZ Label in Intel 8085.
(a) Jump to Label if zero flag is set
(b) Jump to Label if zero flag is not set
(c) Jump to Label
(d) None of the above

56. A superkey such that set of its attributes (one or more than one) does not form a superkey is called
(a) candidate key (b) primary key
(c) foreign key (d) None of these

57. Six students A, B, C, D, E and F are sitting in the field. A and B are from Delhi while the rest are from Bengaluru. D and F are tall while others are short. A, C and D and girls while others are boys. Which is the tall girl form Bengaluru?
(a) C (b) D
(c) E (d) F

58. The value of the integral $\int_0^1 x^{-1/2} dx$ can be found by using
(a) Trapezoidal rule (b) Simpson's rule
(c) Mid-point rule (d) All of these

59. Consider the normalized floating point number in base b so that mantissa X satisfies the condition $(1/b) \xi X < 1$. Experience shows that X has the following probability density function $f_x(x) = k/x, k > 0$. The value of k is
(a) 1 (b) $\ln \beta$
(c) $1/\ln \beta$ (d) None of these

60. Find the missing character from among the given alternatives.

?	1	2
21	22	40
1	2	5
20	23	43

(a) 5 (b) 4
(c) 3 (d) 2

61. If $f(x+y) = f(x)+f(y)-xy-1$ for all x, y and $f(1) = 1$, then the number of solutions of $f(n) = n, n \in N$, is
(a) one (b) two
(c) four (d) None of these

62. $\lim_{n \to \infty} \left(\frac{1}{1-n^2} + \frac{2}{1-n^2} + \ldots + \frac{n}{1-n^2} \right)$ is equal to
(a) 0 (b) $-1/2$
(c) 1/2 (d) None of these

63. If the line joining the points (0, 3) and (5, -2) is a tangent to the curve $y = c/(x+1)$, then the value of c is
(a) 1 (b) -2
(c) 4 (d) None of these

64. Let $f(x)$ be a continuous function such that $f(a-x) + f(x) = 0$ for all $x \in [0, a]$. Then, $\int_0^a \frac{dx}{1+e^{f(x)}}$ is equal to
(a) a (b) $a/2$
(c) $f(a)$ (d) $f(a)/2$

65. The degree of the differential equation satisfying $a(x-y) = \sqrt{1-x^2} + \sqrt{1+y^2}$
(a) 1 (b) 2
(c) 3 (d) None of these

66. A particle in equilibrium is subjected to four forces $F_1 = -10\mathbf{k}$, $F_2 = \frac{u}{13}(4\mathbf{i} - 12\mathbf{j} + 3\mathbf{k})$, $F_3 = \frac{v}{13}(-4\mathbf{i} - 12\mathbf{j} + 3\mathbf{k})$ and $F_4 = w(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$. The value of $u + v + w$ is given by
(a) $40 \operatorname{cosec}\theta + 130/3$
(b) $40 \operatorname{cosec}\theta + 130$
(c) $130 \operatorname{cosec}\theta + 130$
(d) None of the above

67. If $1, \omega, \omega^2, \ldots, \omega^{n-1}$ are the nth roots of unity, then $(2-\omega)(2-\omega^2)\ldots(2-\omega^{n-1})$ equals to
(a) $2^n - 1$
(b) ${}^nC_1 + {}^nC_2 + \ldots + {}^nC_n$
(c) $[{}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots + {}^{2n+1}C_n]^{1/2} - 1$
(d) None of the above

68. If x is a real and $k = (x^2 - x + 1)/(x^2 + x + 1)$, then
(a) $(1/3) \le k \le 3$ (b) $k \ge 5$
(c) $k \le 0$ (d) None of these

69. If $\cot\alpha + \tan\alpha = m$ and $\frac{1}{\cos\alpha} - \cos\alpha = n$, then
(a) $m(mn^2)^{1/3} - n(nm^2)^{1/3} = 1$
(b) $m(nm^2)^{1/3} - n(mn^2)^{1/3} = 1$
(c) $n(mn^2)^{1/3} - m(nm^2)^{1/3} = 1$
(d) $n(nm^2)^{1/3} - m(mn^2)^{1/3} = 1$

70. In a pile of reading material, there are novels, story, books, dramas and comics. Every novels have a drama next to it, every story book has a comic next to it and there is no story book next to a novel. If there be a novel at the top and the number of books be 40, the order of the books in the pile is
(a) nscd (b) ndsc
(c) csdn (d) dncs

71. Each side of an equilateral triangle subtends an angle of 60° at the top of a tower h metre high located at the centre of the triangle. If a is the length of each side of the triangle, then
(a) $3a^2 = 2h^2$ (b) $2a^2 = 3h^2$
(c) $a^2 = 3h^2$ (d) $3a^2 = h^2$

72. The lines $x - 2y - 6 = 0$, $3x + y - 4 = 0$ and $\lambda x + 4y + \lambda^2 = 0$ are concurrent, if λ is equal to
(a) 2 (b) -3
(c) 4 (d) None of these

73. If the line $y = mx$ is one of the bisectors of the lines $x^2 - y^2 + 4xy = 0$, then the value of m is given by

(a) $m = 1$ (b) $m^2 - m = 0$
(c) $m^2 + m - 1 = 0$ (d) None of these

74. What would be the sequence of nodes in post order traversal of a binary tree whose inorder and preorder traversals are as under?

Inorder	C	D	E	B	A
Preorder	A	B	C	D	E

(a) DEBCA
(b) EDCAB
(c) EDCBA
(d) EDBCA

75. The output of the following program will be

```
#include <stdio.h>
main ( )
{
char ch[10];
int i;
for (i = 0; i < 9; i++)
(ch+i) = 65;
*(ch+i) = '\0';
printf("n%s", ch);
}
```

(a) AAAAAAAAA
(b) BBBBBBBBB
(c) 656565656565656565
(d) None of the above

76. The number of flip-flops required to design decade counter is

(a) 3 (b) 4
(c) 5 (d) 10

77. The instruction LDA 2000H in Intel 8085.

(a) Loads data from memory location 2000H to register A
(b) Loads data from memory location 2000H to register B
(c) Loads data from memory location 2000H to register C
(d) Loads data from memory location 2000H to register D

78. The degree of the Cartesian product of two relations P and Q is given by

(a) $|P| * |Q|$ (b) $|P| + |Q|$
(c) $\max(|P|, |Q|)$ (d) None of these

79. Let $x_1, x_2, \ldots, x_n$ be a random sample drawn from normal population with mean m and variance s^2. Writing

$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ and $s^2 = \frac{\sum_{i=1}^{n} (x_i - x)^2}{n-1}$ the statistics $\frac{(\bar{x} - \mu)}{s/\sqrt{n}}$

follows

(a) t-distribution
(b) normal distribution
(c) chi-square distribution
(d) F-distribution

80. If point is in motion on the curve $12y = x^3$, then ordinate is changing at a faster rate than the abscissa in the interval

(a) $(-2, 2)$ (b) $(-\infty, -2) \cup (-2, \infty)$
(c) $(-2, 0)$ (d) None of these

81. If $\phi(x) = \int \cot^4 x \, dx + \frac{1}{3}\cot^3 x - \cot x$ and $\phi\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$, then $\phi(x)$ is

(a) $\pi - x$ (b) $-\pi + x$
(c) $\pi/2 - x$ (d) None of these

82. In a cricket match, five batsmen A, B, C, D and E scored an average of 36 runs. D scored 5 more than E; E scored 8 fewer than A; B scored as many as D and E combined; and B and C scored 107 between them. How many runs did E score?

(a) 20 (b) 45
(c) 28 (d) 62

83. Let $f(x) = ax^2 + bx + c$; $a, b, c \in R$ and $a \neq 0$. Suppose, $f(x) > 0$ for all $x \in R$. Let $g(x) = f(x) + f'(x) + f''(x)$. Then,

(a) $g(x) > 0$ for all $x \in R$ (b) $g(x) < 0$ for all $x \in R$
(c) $g(x) = 0$ for real roots (d) None of these

84. The value of the determinant

$$\begin{vmatrix} 1 & a & a^2 \\ \cos(n-1)x & \cos nx & \cos(n+1)x \\ \sin(n-1)x & \sin nx & \sin(n+1)x \end{vmatrix}$$ is zero, if

(a) $\sin x = 0$ (b) $\cos x = 0$
(c) $a = 0$ (d) None of these

85. Two of the straight lines given by $3x^3 + 3x^2y - 3xy^2 + my^3 = 0$ are at right angles, if

(a) $m = -1/3$ (b) $m = 1/3$
(c) $m = -3$ (d) $m = 3$

86. Assume that an upper triangular matrix $a[0 \ldots n-1, 0 \ldots n-1]$ is stored in a linear array $b[0 \ldots (n(n+1)/2 - 1]$ is **lexicographical** order. If $a[0, 0]$ is stored in $b[0]$, where is $a[30, 40]$ stored in b array for $n = 50$?

(a) b [1020] (b) b [1076]
(c) b [1075] (d) b [1074]

87. Which of the following statements is/are true?

(a) Structures can be compared using ==
(b) Unions can be compared using == to determine if they are equal
(c) Structures are always passed to function by reference
(d) All of the above

88. Which of the following shift registers will result in fast data transmission?

(a) Serial in parallel out
(b) Parallel in serial out
(c) Parallel in parallel out
(d) Serial in serial out

89. The simplified Boolean function for $F(x, y, z) = \Sigma(0, 2, 3, 4, 5, 6)$ is

(a) $x\bar{y} + \bar{x}y + \bar{z}$ (b) $x\bar{y} + \bar{x}y + \bar{z}\bar{y}$
(c) $x\bar{y} + \bar{x}y + zy$ (d) None of these

90. Let $X = BCD$ and X under $F = \{A \to BC, CD \to E, E \to C, D \to AEH, ABH \to BD, DH \to BC\}$. Then, X^+ of X under F is given by

(a) $ABCD$ (b) $ABEH$
(c) $CDEH$ (d) $ABCDEH$

91. Subway trains on a certain line run every half hour between midnight and six in the morning. Find the probability that a person entering the station at a random time during this period will have to wait atleast twenty minutes.
(a) 1/2 (b) 2/3
(c) 1/3 (d) 1/6

92. X is a random variable with pdf $f(x) = 1/2a, -a < x < a \cdot E(e^{tX})$ equals to
(a) $\sinh(at)/at$ (b) e^{at}
(c) $e^{at} - e^{-at}$ (d) $\cosh(at)/at$

93. The value of $\lim_{x \to 1} \sin^{-1}\left(\log_3 \frac{x}{3}\right)$ is equal to
(a) $-\pi/2$ (b) $\pi/2$
(c) 0 (d) None of these

94. If there is an error of k% is measuring the edge of a cube, then the per cent error in estimating its volume is
(a) k (b) $3k$ (c) $k/3$ (d) k^3

95. The solution of differential equation $(x-y)^2 \frac{dy}{dx} = a^2$ is
(a) $y = \frac{a}{2} \log \left|\frac{x-y-a}{x-y+a}\right| + C$
(b) $x = \frac{a}{2} \log \left|\frac{x-y-a}{x-y+a}\right| + C$
(c) $y^2 = a \log \left|\frac{x-y+a}{x-y-a}\right| + C$
(d) None of the above

96. If **a** and **b** are two unit vectors, then the vector $(\mathbf{a}+\mathbf{b}) \times (\mathbf{a} \times \mathbf{b})$ is parallel to the vector
(a) a − b (b) a + b
(c) 2a − b (d) 2a + b

97. In a ΔABC, if $\tan(A/2) = 5/6$ and $\tan(B/2) = 20/37$, the sides a, b and c are in
(a) AP (b) GP
(c) HP (d) None of these

98. The equation of the circle through (1, 1) and the points of intersection of $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$, is
(a) $4x^2 + 4y^2 - 30x - 10y - 32 = 0$
(b) $4x^2 + 4y^2 + 30x - 13y - 25 = 0$
(c) $4x^2 + 4y^2 + 30x - 13y + 25 = 0$
(d) None of the above

99. What will be the output of the following program?
```
String s1, s2;
if (strcmp (s1,s2))
printf("strings are equal");
```
(a) Does not print anything
(b) Output will be the strings are equal
(c) Gives syntax error
(d) Gives unpredictable output

100. The minimum number of nodes in an AVL (Height Balanced binary tree) of height 6 is
(a) 20 (b) 33
(c) 24 (d) 36

101. What is the extension of output of the Compiler?
(a) .obj (b) .asm
(c) .exe (d) .c

102. The sum of the income of A and B is more than that of C and D taken together. The sum of the income of A and C is the same as that of B and D taken together. Moreover, A earns half as much as the sum of the income of B and D. Which of the following statements is not correct?
(a) A earns more than B
(b) B earns more than D
(c) B earns more than C
(d) A earns same as C

103. The interval which contains the eigen values of the symmetric matrix.
$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 3 \end{bmatrix} \text{ is}$$
(a) $(2, \infty)$ (b) $(-1, 9)$
(c) $(1, 8)$ (d) $(-1, 7)$

104. The train for Kanpur leaves every two and a half hours from New Delhi Railway Station. An announcement was made at the station that the train for Kanpur had left 40 min ago and the next train will leave at 18:00 h. At what time was the announcement made?
(a) 15 : 30 h (b) 17 : 10 h
(c) 16 : 00 h (d) None of these

105. If $\frac{a_0}{n+1} + \frac{a_1}{n} + \frac{a_2}{n-1} + \ldots + \frac{a_{n-1}}{2} + a_n = 0$, then the function $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n$ has in (0, 1)
(a) atleast one zero (b) atmost one zero
(c) only 3 zeros (d) only 2 zeros

106. $\int x\, e^{x^2} \cos(e^{x^2})\, dx$ is equal to
(a) $2\sin(e^{x^2}) + C$ (b) $\sin(e^{x^2}) + C$
(c) $\frac{1}{2}\cos(e^{x^2}) + C$ (d) $\frac{1}{2}\sin(e^{x^2}) + C$

107. In a certain office, 1/3 of the workers are women, 1/2 of the women are married and 1/3 of the married women have children. If 3/4 of the men are married and 2/3 of the married men have children, what part of workers are without children?
(a) 5/18 (b) 4/9
(c) 11/18 (d) 17/36

108. If $\frac{\tan\alpha - i(\sin(\alpha/2) + \cos(\alpha/2))}{1 + 2i\sin(\alpha/2)}$ is purely imaginary, then α is not given by
(a) $n\pi + \pi/4$ (b) $n\pi - \pi/4$
(c) $2n\pi$ (d) $2n\pi + \pi/4$

109. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$, $(abc \neq 0)$ is equal to the sum of the squares of their reciprocals, then $a/c, b/a, c/b$ are in
(a) arithmetic progression
(b) geometric progression
(c) harmonic progression
(d) None of the above

110. The line $y = x + 5$ does not touch
(a) the parabola $y^2 = 20x$
(b) the ellipse $9x^2 + 16y^2 = 144$
(c) the hyperbola $4x^2 - 29y^2 = 116$
(d) the circle $x^2 + y^2 = 25$

111. In the following C fragment with reference to i and j, which one of the following statements is true?
int x; int *i = & x ; int *j = & x ;
(a) i and j are overloaded
(b) i and j exhibit polymorphism
(c) i and j are aliases
(d) Value of i and j are always equal

112. I. All children are inquisitive.
II. Some children are inquisitive.
III. No children are inquisitive.
IV. Some children are not inquisitive.
Find out which two statements cannot be true simultaneously, but can both be false.
(a) I and III (b) II and III
(c) I and IV (d) III and IV

113. A cube is coloured in such a way that each pair of its adjacent sides have the same colour. What is the minimum number of colours you require?
(a) 2 (b) 3
(c) 4 (d) None of these

114. Ravi is not wearing white and Ajay is not wearing blue. Ravi and Sohan wear different colours. Sachin alone wears red. What is the Sohan's colour, if all four of them are wearing different colours?
(a) Red (b) Blue
(c) White (d) Can't say

115. The maximum value of the step-size h that can be used in the tabulation of $f(x) = \sin(x)$ in the interval [1, 3] so that the error in linear interpolation is less than equal to 1.25×10^{-7}, is
(a) 0.1 (b) 0.01
(c) 0.001 (d) 0.0001

116. We define $\binom{r}{k} = \frac{r(r-1)\dots(r-k+1)}{k(k-1)\dots 1}$, when k is non-negative and $\binom{r}{k} = 0$, when k is negative. Thus, $\binom{-7.2}{2}$ equals to
(a) 0 (b) 29.52
(c) 1.52 (d) ∞

117. Suppose, the random variable X has the density function
$$f(x) = \begin{cases} (1+\lambda)x^{\lambda}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$
The maximum likelihood estimate of λ based on a given random sample $X_1 = x_1, X_2 = x_2, \dots, X_n = x_n$, is
(a) $\lambda = -\frac{n}{\sum_{i=1}^{n} x_i}$ (b) $\lambda = -\frac{n}{\sum_{i=1}^{n} \ln x_i}$
(c) $\lambda = -1 - \frac{n}{\sum_{i=1}^{n} x_i}$ (d) $\lambda = -1 - \frac{n}{\sum_{i=1}^{n} \ln x_i}$

118. The function g defined for all real x by $g(x) = e^x - 1 - x$ has a minimum value
(a) -5 (b) -3
(c) 0 (d) 1

119. Suppose, as interactive computer system is proposed for which it is estimated that the mean response time $E(T) = 0.5$ s and standard deviation $\sigma = 0.1$ s. Using Chebyshev's inequality, the probability $P[|T - 0.5| \geq 0.25]$ is
(a) 0.16 (b) 0.84
(c) 0.25 (d) 0.75

120. X is a normal variate with μ and σ^2. The value of $E(e^X)$ is
(a) e^{μ}
(b) $e^{\mu + \sigma}$
(c) $e^{\mu + \sigma^2/2}$
(d) $(e^{\mu + \sigma})^2$

Answers *with* Solutions

1. (a) $X_{n+1} = X_n + 2X_{n-1}$
$\Rightarrow \quad X_{n+1} - X_n - 2X_{n-1} = 0$
$\Rightarrow \quad x^2 - x - 2 = 0$
$\Rightarrow \quad (x-2)(x+1) = 0$
$\Rightarrow \quad x = -1, 2$
$\Rightarrow \quad X_n = C_1(-1)^n + C_2(2)^n$
$X_0 = 1 = C_1 + C_2$ and $X_1 = 1 = -C_1 + 2C_2$
$\Rightarrow \quad C_2 = 2/3; C_1 = 1/3$
$\Rightarrow \quad X_n = \frac{1}{3}[(2)^{n+1} + (-1)^n]$ ...(i)
$Y_{n+1} = 2Y_n + 3Y_{n-1}$
$\Rightarrow \quad x^2 - 2x - 3 = 0$
$\Rightarrow \quad x = 3, -1$
$\Rightarrow \quad Y_n = C_1(3)^n + C_2(-1)^n;$
$Y_0 = 1 = C_1 + C_2; Y_1 = 3C_1 - C_2 = 7$
$\Rightarrow \quad C_1 = 2, C_2 = -1$
$\Rightarrow \quad Y_n = 2(3)^n + (-1)^{n+1}$ ...(ii)
From Eqs. (i) and (ii), we see that there is only one term is common, when $n = 0$.

2. (b) $\text{cov}(I_A, I_B) = E(I_A, I_B) - E(I_A)E(I_B)$
$= 1 \cdot 1\, P(AB) - 1P(A)1P(B)$
$= P(AB) - P(A)P(B)$

3. (c) $I = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} |x - \mu| e^{-(x-\mu)^2/2\sigma^2} dx$
Let $\frac{x - \mu}{\sqrt{2}\sigma} = y$
$\Rightarrow \quad \frac{dx}{\sqrt{2}\sigma} = dy$

$\Rightarrow \quad I = \frac{\sqrt{2}\sigma}{\sqrt{\pi}}\int_{-\infty}^{\infty} |y| e^{-y^2} dy$

$= \frac{2\sqrt{2}\sigma}{\sqrt{\pi}}\int_0^{\infty} y e^{-y^2} dy$ (even function)

Let $y^2 = z$

$\Rightarrow \quad 2y dy = dz$

$\Rightarrow \quad dy = dz/2\sqrt{z}$

$\Rightarrow \quad I = \frac{\sqrt{2}\sigma}{\sqrt{\pi}}\int_0^{\infty} e^{-z} dz = \sqrt{\frac{2}{\pi}}\sigma = \left(\frac{2}{\pi}\right)^{1/2}\sigma$

$\left(\because \int_0^{\infty} e^{-z} z^{1-1} dz = \pi = 1\right)$

4. (a) $\mathbf{a} = 3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$; $\mathbf{b} = 4\mathbf{i} - \mathbf{j} + 3\mathbf{k}$

Third side $\mathbf{c}$ is $\mathbf{a} - \mathbf{b} = -\mathbf{i} + 7\mathbf{j} - 5\mathbf{k}$

If θ is angle between a and c, then

$\mathbf{a}\cdot\mathbf{c} = |\mathbf{a}||\mathbf{c}|\cos\theta$

$\Rightarrow \quad -3 + 42 + 10 = 7\sqrt{75}\cos\theta$

$\Rightarrow \quad \cos\theta = \frac{7}{\sqrt{75}} \Rightarrow \theta = \cos^{-1}\frac{7}{\sqrt{75}}$

5. (d) $I = \int \frac{\log x^2}{x} dx = \int 2\frac{\log x}{x} dx$

Let $\log x = y \Rightarrow \frac{1}{x}dx = dy$

$\Rightarrow \quad I = \int 2y dy = y^2 + C = (\log x)^2 + C$

$= \frac{(2\log x)^2}{4} + C$

$= \frac{(\log x^2)^2}{4} + C$

6. (c) $P(X_1 + X_2 \geq 1) = 1 - P(X_1 + X_2 = 0)$

$= 1 - P(X_1 = 0)\, P(X_2 = 0)$

$= 1 - {}^3C_0(2/3)^3 \cdot {}^5C_0(2/3)^5$

$= 1 - (2/3)^8$

7. (a) Array is used for aggregation of data using Cartesian product through disjunction of its field.

8. (b) $\tan A \tan B$ will be maximum, when $A = B = \frac{\pi}{6}$

$\Rightarrow \quad \frac{1}{\sqrt{3}}\cdot\frac{1}{\sqrt{3}} = \frac{1}{3}$

9. (b) $I = \int x\tan^{-1}x\, dx = (\tan^{-1}x)\frac{x^2}{2} - \int \frac{x^2}{2(1+x^2)}dx$

$= \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right)dx$

$= \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}x + \frac{1}{2}\tan^{-1}x + C$

$= \frac{(x^2+1)\tan^{-1}x - x}{2} + C$

10. (d) Given, $\mathbf{d} = \lambda(\mathbf{a}\times\mathbf{b}) + \mu(\mathbf{b}\times\mathbf{c}) + \eta(\mathbf{c}\times\mathbf{a})$ and $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}] = 1/8$

$\mathbf{c}\cdot\mathbf{d} = \lambda\mathbf{c}\cdot(\mathbf{a}\times\mathbf{b}) + \mu\,\mathbf{c}\cdot(\mathbf{b}\times\mathbf{c}) + \eta\,\mathbf{c}\cdot(\mathbf{c}\times\mathbf{a})$

$= \lambda[\mathbf{a}\ \mathbf{b}\ \mathbf{c}] + \mu[\mathbf{c}\ \mathbf{b}\ \mathbf{c}] + \eta[\mathbf{c}\ \mathbf{c}\ \mathbf{a}]$

$= \lambda\cdot 1/8 + \mu\cdot 0 + \eta\cdot 0$

$= \lambda/8$

$\Rightarrow \quad \mathbf{c}\cdot\mathbf{d} = \frac{1}{8}\lambda$;

Similarly, $\mathbf{b}\cdot\mathbf{d} = \frac{1}{8}\eta$; $\mathbf{a}\cdot\mathbf{d} = \frac{1}{8}\mu$

$\Rightarrow \quad (\mathbf{a}+\mathbf{b}+\mathbf{c})\cdot\mathbf{d} = \frac{1}{8}(\lambda+\eta+\mu)$

$\Rightarrow \quad \lambda+\eta+\mu = 8(\mathbf{a}+\mathbf{b}+\mathbf{c})\cdot\mathbf{d}$

11. (d) Let given vertices are A, B, C and D.

$\Rightarrow \quad \mathbf{AB} = -2\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$

$\mathbf{BC} = 6\mathbf{i} + 2\mathbf{j} + (\lambda - 7)\mathbf{k}$

$\mathbf{CD} = 2\mathbf{i} - 3\mathbf{j} + (7 - \lambda)\mathbf{k}$

$V = 11$ (given)

Volume of tetrahedron

$\Rightarrow \quad \frac{1}{6}[\mathbf{AB}\ \mathbf{BC}\ \mathbf{CD}] = 11$

$\Rightarrow \quad \begin{vmatrix} -2 & 3 & -3 \\ 6 & 2 & \lambda-7 \\ 2 & -3 & 7-\lambda \end{vmatrix} = 66$

Expand w.r.t. R_1

$\Rightarrow \quad -2(14 - 2\lambda + 3\lambda - 21)$

$- 3(42 - 6\lambda - 2\lambda + 14) - 3(-18 - 4) = 66$

$\Rightarrow \quad -2\lambda + 14 + 24\lambda - 168 + 66 = 66$

$\Rightarrow \quad 22\lambda = 154$

$\lambda = 7$

12. (d) $z^{1/3} = a - ib \Rightarrow z = (a - ib)^3$

$\Rightarrow \quad x + iy = a^3 + ib^3 - 3iab(a - ib)$

$= a^3 - 3ab^2 + i(b^3 - 3a^2b)$

$\Rightarrow \quad x = a(a^2 - 3b^2)$

and $\quad y = b(b^2 - 3a^2)$

$\Rightarrow \quad bx - ay = ab(a^2 - 3b^2 - b^2 + 3a^2)$

$= 4ab(a^2 - b^2) = kab(a^2 - b^2)$

$\Rightarrow \quad k = 4$

13. (d) By using $\sum_{k=1}^{n-1}\sin\frac{2k\pi}{n} = 0$,

$\sum_{k=1}^{n-1}\cos\frac{2k\pi}{n} = -1$

Put $n = 11$

$\sum_{k=1}^{10}\sin\frac{2k\pi}{11} = 0, \quad \sum_{k=1}^{10}\cos\frac{2k\pi}{11} = -1$

$\Rightarrow \quad \sum_{k=1}^{10}\left(\sin\frac{2\pi k}{11} - i\cos\frac{2\pi k}{11}\right) = 0 - i(-1) = i$

Alternate method

$= \sum_{k=1}^{10}\left\{\sin\frac{2\pi k}{11} - i\cos\frac{2\pi k}{11}\right\}$

$= -i\sum_{k=1}^{10}\left\{\cos\frac{2\pi k}{11} - i\sin\frac{2\pi k}{11}\right\}$

$= -i\sum_{k=1}^{10} e^{-i\frac{2\pi k}{11}}$

$= -i\cdot e^{-i\frac{2\pi}{11}(1+2+3+\ldots\ldots+10)}$

$= -i\cdot e^{-i\frac{2\pi}{11}\left\{\frac{10\times 11}{2}\right\}}$

$= -i\cdot e^{-i10\pi}$

$= -i\{\cos 10\pi - i\sin 10\pi\}$

$= -i(1 - i\cdot 0) = -i$

14. (c) Product of roots < 0

$\Rightarrow \quad \frac{a^2 - 3a + 2}{3} < 0$

$\Rightarrow \quad a^2 - 3a + 2 < 0$

$\Rightarrow \quad (a-1)(a-2) < 0 \Rightarrow 1 < a < 2$

15. (b) $\cos 2x + a\sin x = 2a - 7$

$\Rightarrow \quad 1 - 2\sin^2 x + a\sin x = 2a - 7$

$\Rightarrow \quad 2\sin^2 x - a\sin x + 2(a-4) = 0$

$\Rightarrow \quad \sin x = \dfrac{a \pm \sqrt{a^2 - 16(a-4)}}{4} = \dfrac{a \pm (a-8)}{4}$

$= \dfrac{a-4}{2}$ or 2

$\Rightarrow \quad \sin x = \dfrac{a-4}{2} \qquad (\sin x \neq 2)$

$\Rightarrow \quad -1 \leq \dfrac{a-4}{2} \leq 1$

$\Rightarrow \quad -2 \leq a - 4 \leq 2$

$\Rightarrow \quad 2 \leq a \leq 6$

16. (c) $x = \sum_{n=0}^{\infty} \cos^{2n} a = 1 + \cos^2 a + \cos^4 a + \dots$

$= 1/1 - \cos^2 a$

$\Rightarrow \quad x = \operatorname{cosec}^2 a \Rightarrow \sin^2 a = \dfrac{1}{x}$

$y = \sum_{n=0}^{\infty} \sin^{2n} a = 1/1 - \sin^2 a = \sec^2 a$

$\Rightarrow \quad \cos^2 a = \dfrac{1}{y}$

$z = \sum_{n=0}^{\infty} \cos^{2n} a \sin^{2n} a = 1/1 - \cos^2 a \sin^2 a$

$\Rightarrow \quad z = \dfrac{1}{1 - \dfrac{1}{xy}} = \dfrac{xy}{xy - 1}$

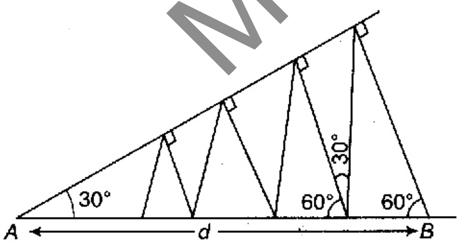
$\Rightarrow \quad xyz - z = xy$

$\Rightarrow \quad xyz = xy + z$

17. (d) Total length $= d\sin 30° + d\sin 30°\sin 60° + d\sin 30°\sin 60°\sin 30° + \dots$

$= \dfrac{d}{2} + \dfrac{\sqrt{3}}{2}\cdot\dfrac{d}{2} + \dfrac{\sqrt{3}d}{2^3} + \dfrac{(\sqrt{3})^2 d}{2^4} + \dots$

$= \left(\dfrac{d}{2} + \dfrac{\sqrt{3}d}{2^3} + \dots\right) + \left(\dfrac{\sqrt{3}}{2}\cdot\dfrac{d}{2} + \dfrac{(\sqrt{3})^2 d}{2^4} + \dots\dots\right)$



$= \dfrac{d}{2}\left[1 + \dfrac{\sqrt{3}}{4} + \left(\dfrac{\sqrt{3}}{4}\right)^2 + \dots\right] + \dfrac{\sqrt{3}d}{4}\left[1 + \dfrac{\sqrt{3}}{4} + \left(\dfrac{\sqrt{3}}{4}\right)^2 + \dots\right]$

$= \dfrac{d}{2}\left[1 + \dfrac{\sqrt{3}}{2}\right] \Big/ 1 - \dfrac{\sqrt{3}}{4}$

$= \dfrac{d(2+\sqrt{3})}{4 - \sqrt{3}}$

18. (a) $\cos^2(A-B) + \cos^2 B - 2\cos(A-B)\cos A\cos B$

$= \cos^2(A-B) + \cos^2 B - \cos(A-B) \times [\cos(A-B) + \cos(A+B)]$

$= \cos^2 B - \cos(A-B)\cdot\cos(A+B)$

$= \cos^2 B - [\cos^2 A - \sin^2 B] = 1 - \cos^2 A = \sin^2 A$

19. (d) $x = \tan 3A \cot A$

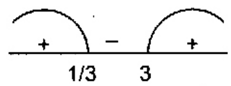
$= \dfrac{(3\tan A - \tan^3 A)\cot A}{1 - 3\tan^2 A}$

$\Rightarrow \quad x = \dfrac{3 - \tan^2 A}{1 - 3\tan^2 A} \Rightarrow x - 3x\tan^2 A = 3 - \tan^2 A$

$\Rightarrow \quad (1 - 3x)\tan^2 A = 3 - x$

$\Rightarrow \quad \tan^2 A = \dfrac{3-x}{1-3x} \geq 0$

$\Rightarrow \quad \dfrac{(x-3)(3x-1)}{(3x-1)^2} \geq 0$



$\Rightarrow \quad x < 1/3$ or $x \geq 3$

20. (b) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B} = \dfrac{\dfrac{5}{6} + \dfrac{1}{11}}{1 - \dfrac{5}{66}} = 1$

$\Rightarrow \quad A + B = \pi/4$

21. (c)

22. (a) $(756.603)_8 = (\_\_\_\_)_{16}$

Firstly, convert $(756.603)_8$ into decimal

7 5 6 · 6 0 3

$7 \times 8^2 + 5 \times 8^1 + 6 \times 8^0 + 8^{-1} \times 6 + 8^{-2} \times 0 + 8^{-3} \times 3$

$\Rightarrow \quad 448 + 40 + 6 + 0.75 + 0.00585938$

$\Rightarrow \quad (494.75585938)_{10}$

Now, convert this decimal form into binary.

$\Rightarrow$

2	494	
2	247	0
2	123	1
2	61	1
2	30	1
2	15	0
2	7	1
2	3	1
	1	1

So, $(494)_{10} = (111101110)_2$

$0.75585938 \times 2 = 1.51171876 \quad 1$

$0.51171876 \times 2 = 1.02343752 \quad 1$

$0.02343752 \times 2 = 0.04687504 \quad 0$

$0.04687504 \times 2 = 0.09375008 \quad 0$

$0.09375008 \times 2 = 0.18750016 \quad 0$

$0.18750016 \times 2 = 0.37500032 \quad 0$

$0.37500032 \times 2 = 0.75000064 \quad 0$

$0.75000064 \times 2 = 1.50000128 \quad 1$

$0.50000128 \times 2 = 1.00000256 \quad 1$

$0.00000256 \times 2 = 0.00000512 \quad 0$

$0.00000512 \times 2 = 0.00001024 \quad 0$

$0.00001024 \times 2 = 0.00002048 \quad 0$

$\therefore (0.75585938)_{10} = (0.110000011000)_2$

Now, convert these binary digits into hexadecimal form

$\because (494.75585938)_{10} = (111101110.110000011000)_2$

$= (1EE.C18)_{16}$

23. (c) As the angles are in AP, so angles, will be $60° - \theta$, $60°$ and $60° + \theta$. If the length of the third side is x, then

$\dfrac{10}{\sin(60° + \theta)} = \dfrac{9}{(\sin 60°)} = \dfrac{x}{\sin(60° - \theta)}$ (by sine rule)

$\Rightarrow \quad \sin(60° + \theta) = \dfrac{10\sin 60°}{9} = \dfrac{5\sqrt{3}}{9}$

$\Rightarrow \quad \cos(60° + \theta) = \dfrac{\sqrt{6}}{9}$

$$\sin(60^\circ - \theta) = \sin(120^\circ - (60^\circ + \theta))$$
$$= \sin 120^\circ \cos(60^\circ + \theta) - \cos 120^\circ \sin(60^\circ + \theta)$$
$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{6}}{9} + \frac{1}{2} \cdot \frac{5\sqrt{3}}{9} = \frac{5\sqrt{3} + 3\sqrt{2}}{18}$$
$$\Rightarrow \quad x = \frac{9}{\frac{\sqrt{3}}{2}} \times \frac{5\sqrt{3} + 3\sqrt{2}}{18} = 5 + \sqrt{6}$$

24. (b) Line $a_1x + b_1y + c_1 = 0$ cuts coordinate axes at points
$$\left(-\frac{c_1}{a_1}, 0\right) \text{ and } \left(0, \frac{-c_1}{b_1}\right)$$
Line $a_2x + b_2y + c_2 = 0$ cuts coordinate axes at points
$$\left(\frac{-c_2}{a_2}, 0\right) \text{ and } \left(0, \frac{-c_2}{b_2}\right)$$
These four points will be concyclic, if
$$\left(\frac{-c_1}{a_1}\right)\left(\frac{-c_2}{a_2}\right) = \left(\frac{-c_1}{b_1}\right)\left(\frac{-c_2}{b_2}\right)$$
$$a_1a_2 - b_1b_2 = 0$$

25. (d) All the three statements are true

I. Static languages do not support recursion. Only dynamic languages can support recursion.

II. The memory requirement for stack-based language such as ALGOL-60 can be estimated at compile time.

III. Resolution of overloaded operators can be done at translation time.

26. (a) The program doesn't work properly because; when we are declaring the array it is containing garbage values. And it would be definitely wrong to send these garbage values to scanfc, as the addresses where it should keep the strings received from the keyboard.

27. (c) Memory = 32 KB
$$= 2^5 \times 2^{10} \text{ B} = 2^{15} \text{ B}$$
So, 15 address lines are required.

28. (a)

29. (c) This is an AP with $a = 201$, $d = 7$

Let there be n terms, then
$$201 + (n-1)7 = 369$$
$$\Rightarrow \quad (n-1)7 = 168$$
$$n = 25$$

30. (d) R(ABCDEH)

F = {A → BC, CD → E, E → C, AH → D}

If we want to find out that any attribute is prime or not, then we have to find out the candidate key for the given relation.

⇒ $\{A^+\} = \{ABC\}$

Closure of A (or $\{A\}^+$) does not contain all the attributes of relation so, it is not the candidate key.

⇒ $\{B\}^+ = \{B\}$ {Not candidate key}

$\{AH\}^+ = \{AHBCDE\}$

{Candidate key is AH because the closure of {AH} contains all the attribute.}

So, A and H attributes are prime attributes.

Hence, option (a) is correct.

In relation, all attributes except A and H like B, C, D, E are non-prime attributes.

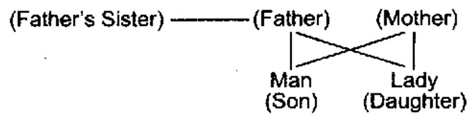
So, option (b) is correct.

AH is the only candidate key possible with this relation.

So, option (c) is correct.

Hence, option (d) is incorrect.

31. (d)



So, lady and man are brother-sister and lady is the sister of the man.

32. (d) $y = x(p + p^3)$ ...(i)

$\left[p = \frac{dy}{dx}\right]$

By differentiating both sides w.r.t. x, we get
$$p = p + p^3 + x(1 + 3p^2)\frac{dp}{dx}$$
$$\Rightarrow \quad -p^3 = x(1 + 3p^2)\frac{dp}{dx}$$
$$\Rightarrow \quad \frac{dx}{x} + \frac{1 + 3p^2}{p^3}dp = 0$$
$$\Rightarrow \int \frac{dx}{x} + 3\int \frac{dp}{p} + \int p^{-3}dp = C$$
$$\Rightarrow \quad \ln x + 3\ln p + \frac{p^{-2}}{-2} = C$$
$$\Rightarrow \quad \ln x + \ln p^3 - \frac{1}{2p^2} = C$$
$$\Rightarrow \quad \ln xp^3 = \frac{1}{2p^2} \quad \text{(taking } C = 0\text{)}$$
$$\Rightarrow \quad xp^3 = e^{\frac{1}{2p^2}}$$
$$\Rightarrow \quad x = \frac{1}{p^3}e^{\frac{1}{2p^2}}$$
$$\Rightarrow \quad y = (p + p^3) \cdot \frac{1}{p^3}e^{\frac{1}{2p^2}} \quad \text{[from Eq. (i)]}$$
$$ye^{-\frac{1}{2p^2}} = 1 + p^{-2}$$

33. (c) $\mathbf{a} \times \mathbf{b} \cdot \mathbf{c} = (|\mathbf{a}||\mathbf{b}|\sin\frac{\pi}{6}\hat{\mathbf{n}}) \cdot \mathbf{c}$
$$= \frac{1}{2}|\mathbf{a}||\mathbf{b}| \quad (\hat{\mathbf{n}} \cdot \mathbf{c} = 1)$$
(Where $\hat{\mathbf{n}}$ and $\mathbf{c}$ are both unit vectors perpendicular to both $\mathbf{a}$ and $\mathbf{b}$)
$$\Rightarrow \quad \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \frac{1}{2}|\mathbf{a}||\mathbf{b}|$$
$$\Rightarrow \quad \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 = \frac{1}{4}|\mathbf{a}|^2|\mathbf{b}|^2$$
$$= \frac{1}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$$

34. (a) Regula–Falsi method do not have rate of convergence as 1

35. (d) Matrix of quadratic form

$Q(x_1, x_2, x_3) = 2x_1^2 + x_2^2 + 3x_3^2 + 2tx_1x_2 + 2x_1x_3$ is
$$\begin{bmatrix} 2 & t & 1 \\ t & 1 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$
It's determinant is, $\Delta = -1 + 3(2 - t^2)$
$$\Rightarrow \quad \Delta = 5 - 3t^2 > 0$$
$$\Rightarrow \quad t^2 < 5/3$$
⇒ $|t| < (5/3)^{1/2}$ is the requirement for positive definite.

36. (b) $f(1) = 1;\ f(n) = 2\sum_{r=1}^{n-1} f(r)$

$\Rightarrow \quad f(2) = 2f(1) = 2$

$\Rightarrow \quad f(3) = 2(1 + 2) = 2 + 2^2 = 6 = 2 \cdot 3$

$\Rightarrow \quad f(4) = 2(1 + 2 + 6) = 18 = 2 \cdot 3^2$

$\Rightarrow \quad f(n) = 2 \times 3^{n-2}$ if $n \ge 2$

$$\sum_{n=1}^{m} f(n) = 1 + 2(1 + 3 + 3^2 + \ldots + 3^{m-2})$$

$$= 1 + 2\frac{(3^{m-1} - 1)}{3 - 1} = 3^{m-1}$$

37. (b) $L = \lim_{x \to 1} \dfrac{\sum_{r=1}^{n} x^r - n}{x - 1}$ $\qquad \left(\dfrac{0}{0}\text{ form}\right)$

By L' Hospital's rule,

$$L = \lim_{x \to i} \sum_{r=1}^{n} r x^{r-1} = \sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$

38. (d) $I = \int_0^{2\pi} \sin x \cos 2x \, dx$

If $\quad f(x) = \sin x \cos 2x$, then

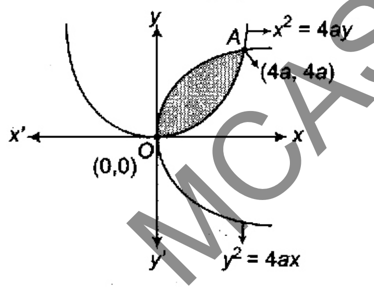
$f(2\pi - x) = (-\sin x)(\cos 2x) = -f(x)$ $\qquad$ (odd function)

$\Rightarrow \quad I = 0$

39. (b) $y^2 = 4ax$ and $x^2 = 4ay$ have point of intersection as $x^4 = 16a^2y^2 = 16a^2(4ax)$

$\Rightarrow \quad x(x^3 - 64a^3) = 0$

$\Rightarrow \quad x = 0, 4a$



So, area included between them will be

$$\int_0^{4a} (y_1 - y_2)dx = \int_0^{4a} \left(\sqrt{4ax} - \frac{x^2}{4a}\right)dx$$

$$= \left[2\sqrt{a}\frac{x^{3/2}}{3/2} - \frac{x^3}{12a}\right]_0^{4a}$$

$$= \frac{4\sqrt{a}}{3}(4a)^{3/2} - \frac{64a^3}{12a} = \frac{32}{3}a^2 - \frac{16}{3}a^2$$

$$= \frac{16a^2}{3}$$

40. (d) $\quad a + b + c = 0$

$\Rightarrow \quad a + b = -c$

$\Rightarrow \quad (a + b)(a + b) = c \cdot c$

$\Rightarrow \quad |a|^2 + |b|^2 + 2a \cdot b = |c|^2$ $\qquad [\because |a| = 3, |b| = 5, |c| = 7]$

$\Rightarrow \quad 9 + 25 + 2|a||b|\cos\theta = 49$

[θ is angle between a and b]

$\Rightarrow \quad 34 + 30\cos\theta = 49$

$\Rightarrow \quad \cos\theta = \dfrac{1}{2}$

$\Rightarrow \quad \theta = 60°$

41. (c) $x_r = \cos\left(\dfrac{\pi}{2^r}\right) + i\sin\left(\dfrac{\pi}{2^r}\right) = e^{i\pi/2^r}$

$$x_1x_2x_3\ldots\infty = e^{i\pi\left(\frac{1}{2} + \frac{1}{2^2} + \ldots\right)} = e^{i\pi\frac{1/2}{1-(1/2)}} = e^{i\pi}$$

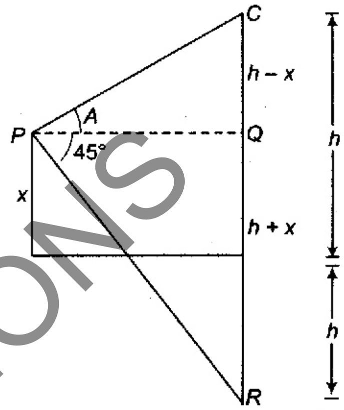
$$= \cos\pi + i\sin\pi = -1$$

42. (c) Word: GIVE; BAT

Code: 5137; 924

So, GATE is coded as 5247.

43. (c)



The above figure is the expression when height of cloud is assumed to be h.

From ΔPQR; $\dfrac{PQ}{h + x} = \cot 45° = 1$

$\Rightarrow \quad PQ = h + x$ $\qquad$ ...(i)

From ΔPQC; $\tan A = \dfrac{h - x}{PQ} = \dfrac{h - x}{h + x}$ $\qquad$ [from Eq. (i)]

$\Rightarrow \quad \dfrac{h + x}{h - x} = \dfrac{1}{\tan A}$

$\Rightarrow \quad \dfrac{h}{x} = \dfrac{1 + \tan A}{1 - \tan A}$

$\Rightarrow \quad h = x\tan(A + 45°)$

44. (c) A, B, C, D are $(6, 3), (-3, 5), (4, -2), (x, 3x)$.

$\Delta ABC = 2\Delta DBC$ $\qquad$ (given)

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 6 & 3 & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{vmatrix} = 2 \times \frac{1}{2}\begin{vmatrix} x & 3x & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 6 & 3 & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2x & 6x & 2 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 6 - 2x & 3 - 6x & -1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad 6 - 2x(7) - (3 - 6x)(-7) - (-14) = 0$

$\Rightarrow \quad 42 - 14x + 21 - 42x + 14 = 0$

$\Rightarrow \quad 56x = 77$

$\Rightarrow \quad x = 11/8$

45. (a) Tangent to parabola $y^2 = 8x$ will be $y = mx + 2/m$. It will make angle of 45° with $y = 3x + 5$, if

$$\tan 45° = \left|\frac{m - 3}{1 + 3m}\right|$$

$\Rightarrow \quad 1 + 3m = |m - 3|$

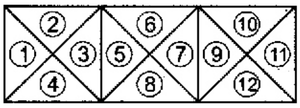
$\Rightarrow \quad m = -2$ or $m = 1/2$

$\Rightarrow \quad$ Tangent is $y = -2x - 1$ or $y = \dfrac{x}{2} + 4$

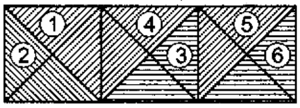
$\Rightarrow \quad 2x + y + 1 = 0$ or $-x + 2y = 8$

46. (c) Bob writes down a number between 1 and 1000. Mary asks the question to Bob and Bob gives the answer by asking 'yes/no' and Bob always tells truth.
This is somewhat called binary searching and complexity
$= \log_2 1000$ ($\because$ complexity $= \log_2 n$; where, $n = 1000$)
$= \dfrac{\log_{10} 10^3}{\log_{10} 2} = \dfrac{3}{\log_{10} 2} = 10$

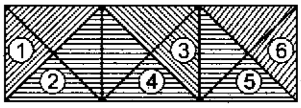
47. (a)



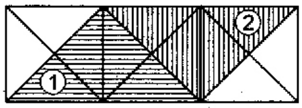
$\Rightarrow$ 12 Triangles



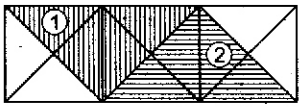
$\Rightarrow$ 6 Triangles



$\Rightarrow$ 6 Triangles

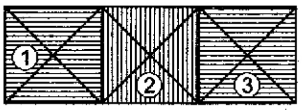


$\Rightarrow$ 2 Triangles

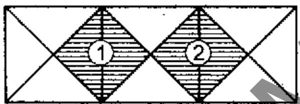


$\Rightarrow$ 2 Triangles

So, there are total 28 triangles.



$\Rightarrow$ 3 Squares



$\Rightarrow$ 2 Squares

So, total 5 squares.

48. (a) AFHO $\rightarrow 1 + 6 + 8 + 15 = 30$
GBDH $\rightarrow 7 + 2 + 4 + 13 = 26$
CHFM $\rightarrow 3 + 8 + 6 + 13 = 30$

$\Rightarrow \dfrac{\text{AFHO}}{\text{GBDM}} = \dfrac{\text{CHFM}}{x} \Rightarrow x = \dfrac{\text{CHFM} \times \text{GBOM}}{\text{AFHO}}$

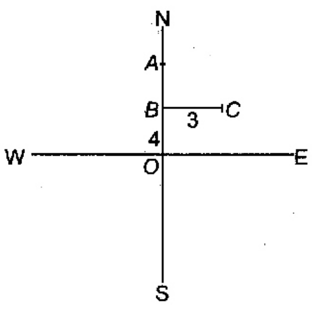
$\Rightarrow \quad x = \dfrac{30 \times 26}{30}\ 26$

$\therefore \quad \text{GBLE} = 7 + 2 + 12 + 5 = 26$

49. (c) Semaphore is the fastest IPC mechanism.

50. (a) Binary normal form is not used to define the syntex rules of a programming language.

51. (d) $OA = 10$



$AB = 6$
$OB = 4$
$BC = 3$
$\Rightarrow \quad OC = \sqrt{4^2 + 3^2}$
$= 5$ km North-East

52. (d) After shifting 4 places towards the right, if position becomes 12th from left earlier position from left would have been 8th, so position from the right will be $21 - 8 + 1 = 14$

53. (d) All the statements are correct.

54. (c) -45 in binary number system is
1 0101101
sign magnitude
It's one complement is 11010010
and two's complement is 11010011.

55. (b) The instruction JNZ Label in Intel 8085 means jump to tabel if zero flag is not set.

56. (a)

57. (b) There are six students A, B, C, D, E and F

A, B	—	Delhi
C, D, E, F	—	Bengaluru
D, F	—	Tall
A, B, C, E	—	Short
A, C, D	—	Girls
B, E, F	—	Boys

The girls are A, C, D and in these girls, only girl D is tall and D girl is from Bengaluru.

58. (c) $x^{-\frac{1}{2}} = \dfrac{1}{\sqrt{x}} \rightarrow \infty$. As, $x \rightarrow 0$
$\Rightarrow$ Trapezoidal rule and Simpson's rules are ruled out and only mid-point rule will work.

59. (c) $f(x)$ will be probability density function
if $\quad \int_{1/\beta}^{1} f(x)\, dx = 1$
$\Rightarrow \quad \int_{1/\beta}^{1} k/x\, dx = 1$
$\Rightarrow \quad k[\ln x]_{1/\beta}^{1} = 1$
$\Rightarrow \quad k \ln \beta = 1$
$\Rightarrow \quad k = 1/\ln\beta$

60. (d) In each column, sum of first and fourth element is equal to sum of second and third element.
i.e., $1 + 23 = 22 + 2$;
$2 + 43 = 40 + 5$
$\Rightarrow \quad x + 20 = 21 + 1$
$\Rightarrow \quad x = 2$

61. (a) $f(x + y) = f(x) + f(y) - xy - 1$; $f(1) = 1$ gives
$f(2) = f(1 + 1) = f(1) + f(1) - 1 - 1 = 0$
$f(3) = f(2 + 1) = f(2) + f(1) - 2 - 1 = 2$
$\Rightarrow f(n) = n$ is not true for $n = 2, 3$, so there is no solution.
$\Rightarrow f(n) = n$ has only one solution.
i.e., $f(n) - n = 0$ is true only for $n = 1$

62. (b) $\lim\limits_{n \to \infty} \left(\dfrac{1}{1 - n^2} + \dfrac{2}{1 - n^2} + \ldots\ldots + \dfrac{n}{1 - n^2} \right)$

$= \lim\limits_{n \to \infty} \dfrac{n(n + 1)}{2(1 - n^2)} \qquad \left(\because \Sigma n = \dfrac{n(n + 1)}{2} \right)$

$= \lim\limits_{n \to \infty} \dfrac{n^2 + n}{2(1 - n^2)} = \lim\limits_{n \to \infty} \dfrac{1 + \dfrac{1}{n}}{2\left(\dfrac{1}{n^2} - 1 \right)} = -\dfrac{1}{2}$

63. (c) Slope of line joining (0, 3) and (5, -2)
$= \dfrac{-2 - 3}{5 - 0} = -1$

$\Rightarrow$ Equation of line is

$y - 3 = -1(x - 0)$

$\Rightarrow \quad y = 3 - x$

It will be tangent to $y = c/(x + 1)$ if

$$3 - x = \frac{c}{(x + 1)}$$

$\Rightarrow \quad (3 - x)(x + 1) = c$

$\Rightarrow -x^2 + 2x + (3 - c) = 0$ has both roots coincident

$\Rightarrow$ Discriminant $= 0 \Rightarrow B^2 - 4AC = 0$

$\Rightarrow \quad 4 - 4(c - 3) = 0$

$\Rightarrow \quad c = 4$

64. (b) $f(a - x) + f(x) = 0$

$\Rightarrow \quad f(a - x) = -f(x)$ ...(i)

Now,

$$I = \int_0^a \frac{dx}{1 + e^{f(x)}} = \int_0^a \frac{dx}{1 + e^{f(a-x)}}$$

$$= \int_0^a \frac{dx}{1 + e^{-f(x)}} = \int_0^a \frac{e^{f(x)}dx}{1 + e^{f(x)}}, \quad \text{[from Eq.(i)]}$$

$\Rightarrow \quad I + I = \int_0^a dx = a$

$\Rightarrow \quad I = \dfrac{a}{2}$

65. (a) $a(x - y) = \sqrt{1 - x^2} + \sqrt{1 + y^2}$ ...(i)

$$\Rightarrow \quad a\left(1 - \frac{dy}{dx}\right) = -\frac{x}{\sqrt{1 - x^2}} + \frac{y}{\sqrt{1 + y^2}} \cdot \frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{y}{\sqrt{1 + y^2}} + a\right] = a + \frac{x}{\sqrt{1 - x^2}}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{a + \dfrac{x}{\sqrt{1 - x^2}}}{a + \dfrac{y}{\sqrt{1 + y^2}}}$$

Putting the value of a from Eq. (i), we find differential equation of degree one.

66. (a) For equilibrium of forces, F_1, F_2, F_3 and F_4 sum of components along **i**, **j**, **k** will be zero.

i.e., $-10 + \dfrac{3u}{13} + \dfrac{3v}{13} = 0; \ \dfrac{4u}{13} - \dfrac{4v}{13} + w\cos\theta = 0$

and $\quad \dfrac{-12u}{13} - \dfrac{12v}{13} + w\sin\theta = 0$

$\Rightarrow \quad 13w\sin\theta = 12(u + v)$ and $u + v = \dfrac{130}{3}$

$\Rightarrow \quad u + v = \dfrac{13}{12}w\sin\theta = \dfrac{130}{3}$

$\Rightarrow \quad w\sin\theta = 40$ ...(i)

$\Rightarrow u + v + w = \dfrac{130}{3} + w = \dfrac{130}{3} + 40\text{cosec}\,\theta$ [from Eq.(i)]

67. (a) $1, \omega, \omega^2, \ldots\ldots, \omega^{n-1}$ are n roots of unity.

$\Rightarrow (z - 1)(z - \omega)(z - \omega^2) \ldots\ldots (z - \omega^{n-1}) = z^n - 1$

Putting $z = 2$, we get

$(2 - \omega)(2 - \omega^2) \ldots\ldots (2 - \omega^{n-1}) = 2^n - 1$

68. (a) $k = \dfrac{x^2 - x + 1}{x^2 + x + 1}$ (given)

$\Rightarrow \quad x^2(k - 1) + x(k + 1) + (k - 1) = 0$ ...(i)

If x is real, then discriminant of Eq. (i) will be ≥ 0.

$\Rightarrow \quad B^2 - 4AC \geq 0$

$\Rightarrow \quad (k + 1)^2 - 4(k - 1)^2 \geq 0$

$\Rightarrow \quad 3k^2 - 10k + 3 \leq 0$

$\Rightarrow \quad (3k - 1)(k - 3) \leq 0$

$+ \quad - \quad +$

$-\infty \quad 1/3 \quad 3 \quad +\infty$

$\Rightarrow \quad 1/3 \leq k \leq 3$

69. (a) Given, $\cot\alpha + \tan\alpha = m$

$\Rightarrow \quad \dfrac{1}{\cos\alpha\sin\alpha} = m$

$\Rightarrow \quad \sin\alpha\cos\alpha = 1/m$ ....(i)

and $\quad \dfrac{1}{\cos\alpha} - \cos\alpha = n$

$\Rightarrow \quad \dfrac{1 - \cos^2\alpha}{\cos\alpha} = n$

$\Rightarrow \quad \dfrac{\sin^2\alpha}{\cos\alpha} = n \Rightarrow \sin^3\alpha = n\sin\alpha \cdot \cos\alpha$ ...(ii)

$\Rightarrow \quad \sin^3\alpha = n/m$

$\Rightarrow \quad \sin\alpha = (n/m)^{1/3}$

From Eq. (i), $\cos\alpha = \dfrac{1}{m\sin\alpha} = \left(\dfrac{1}{nm^2}\right)^{1/3}$

$\Rightarrow \quad \sec\alpha = (nm^2)^{1/3}$ and $\tan\alpha = (mn^2)^{1/3}$

As, $\quad \sec^2\alpha - \tan 2\alpha = 1$

So, $\quad (nm^2)^{2/3} - (mn^2)^{2/3} = 1$

$\Rightarrow \quad m(mn^2)^{1/3} - n(m^2n)^{1/3} = 1$

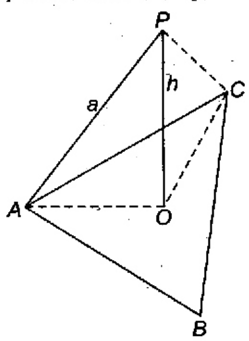
70. (b) These are 40 books of novels, story books, dramas and comics.

$\Rightarrow$ Every novel has a drama next to it *i.e.*, novel-drama, novel-drama, novel-drama ......

$\Rightarrow$ Every story book has a comic next to it. *i.e.*, story book-comic, story book-comic, ...

$\Rightarrow$ There is no story book next to a novel and there be novel at the top, then the order of books will be—(novel, drama, story book, comic) (Top)

71. (b)



If ABC is a triangle and OP is the pole, then as angle of $60°$ is subtended, so $AP = a$, $AO = \dfrac{a}{\sqrt{3}}$.

So, in $\Delta\, AOP$, we have

$(AO)^2 + (OP)^2 = (AP)^2$

$\Rightarrow \quad \dfrac{a}{3} + h^2 = a^2$

$\Rightarrow \quad h^2 = \dfrac{2a^2}{3}$

$\Rightarrow \quad 2a^2 = 3h^2$

72. (a) Lines $x - 2y - 6 = 0$,

$3x + y - 4 = 0$ and

$\lambda x + 4y + \lambda^2 = 0$ will be concurrent, if

$$\begin{vmatrix} 1 & -2 & -6 \\ 3 & 1 & -4 \\ \lambda & 4 & \lambda^2 \end{vmatrix} = 0$$

Expand along R_1

$\Rightarrow (\lambda^2 + 16) + 2(3\lambda^2 + 4\lambda) - 6(12 - \lambda) = 0$

$\Rightarrow 7\lambda^2 + 14\lambda - 56 = 0$

$\Rightarrow \lambda^2 + 2\lambda - 8 = 0$

$\Rightarrow (\lambda + 4)(\lambda - 2) = 0$

$\Rightarrow \lambda = 2, -4$

73. (c) Equation of bisectors of $x^2 - y^2 + 4xy = 0$ is

$$\frac{x^2 - y^2}{1 - (-1)} = \frac{xy}{2} \qquad \left(\because \frac{x^2 - y^2}{a - b} = \frac{xy}{h}\right)$$

$\Rightarrow x^2 - y^2 = xy$

By putting $y = mx$ in Eq. (i), we get

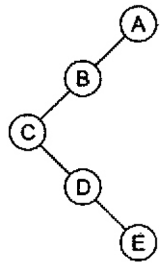
$x^2(1 - m^2) = mx^2$

$\Rightarrow m^2 + m - 1 = 0$

74. (c) Inorder : C D E B A

Preorder : A B C D E

$\Rightarrow$ Tree



$\Rightarrow$ Post order : E D C B A

75. (d) In the given program, an error will come *i.e.*, (L value required)

The statement

(ch + i) = 65;

is not right. We have to write this statement like this

*(ch + i) = 65;

Then, the output will be

AAAAAAAAA

76. (b) To design a decade counter, the required number of flip-flops is 4.

77. (a) The instruction LDA 2000 H in Intel 8085 loads data from memory location 2000 H to register A.

78. (b)

79. (a)

80. (b) 12

$$12\frac{dy}{dt} = 3x^2\frac{dx}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \frac{x^2}{4}\cdot\frac{dx}{dt}$$

$$\Rightarrow \frac{dy}{dt} > \frac{dx}{dt} \qquad \text{(given)}$$

$$\Rightarrow \frac{x^2}{4}\frac{dx}{dt} > \frac{dx}{dt}$$

$$\Rightarrow \frac{x^2}{4} > 1$$

$\Rightarrow x^2 > 4$

$\Rightarrow |x| > 2$

$\Rightarrow x < -2,\ x > 2$

$\Rightarrow x \in (-\infty, -2) \cup (2, \infty)$

81. (d) As, $\int \cot^4 x\,dx = \int \cot^2 x(\operatorname{cosec}^2 x - 1)dx$

$$= \int \cot^2 x \cdot \operatorname{cosec}^2 x\,dx - \int \cot^2 x\,dx$$

$$= -\frac{1}{3}\cot^3 x - \int (\operatorname{cosec}^2 x - 1)\,dx$$

$$= -\frac{1}{3}\cot^3 x + \cot x + x + C \qquad \ldots\text{(i)}$$

$$\Rightarrow \phi(x) = \int \cot^4 x\,dx + \frac{1}{3}\cot^3 x - \cot x$$

$\Rightarrow \phi(x) = x + C \Rightarrow \phi(\pi/2) = \pi/2$

$\Rightarrow C = 0 \Rightarrow \phi(x) = x$

82. (a) $\dfrac{A + B + C + D + E}{5} = 36$

$\Rightarrow A + B + C + D + E = 180 \qquad \ldots\text{(i)}$

$B + C = 107 \qquad \ldots\text{(ii)}$

$\Rightarrow A + D + E = 73 \qquad \ldots\text{(iii)}$

$E = A - 8$

$B = D + E$

$D = E + 5$

From these; $B = 2E + 5$; $A = E + 8$

$A + D + E = 73$

$\Rightarrow 3E + 13 = 73$

$\Rightarrow E = 20$

83. (a) $f(x) = ax^2 + bx + c$

$g(x) = f(x) + f'(x) + f''(x)$

$= ax^2 + bx + c + 2ax + b + 2a$

$= ax^2 + (2a + b)x + (b + c + 2a)$

$f(x) > 0 \Rightarrow a > 0$

and $b^2 - 4ac < 0 \qquad \ldots\text{(i)}$

Now, for $g(x)$; $a > 0$ and Discriminant

$D = (2a + b)^2 - 4a(b + c + 2a) = 4a^2 + b^2 - 4ac - 8a^2$

$= (b^2 - 4ac) - 4a^2 < 0 \qquad$ [from Eq. (i)]

Thus, $g(x) > 0$ for all $x \in R$.

84. (a)
$$\begin{vmatrix} 1 & a & a^2 \\ \cos(n-1)x & \cos nx & \cos(n+1)x \\ \sin(n-1)x & \sin nx & \sin(n+1)x \end{vmatrix} = 0$$

Expanding along R_1

$\Rightarrow 1\{\cos nx \cdot \sin(n+1)x - \cos(n+1)x \cdot \sin nx\}$

$- a\{\cos(n-1)x \cdot \sin(n+1)x - \cos(n+1)x\sin(n+1)x\}$

$+ a^2\{\cos(n-1)x \cdot \sin nx - \cos nx \cdot \sin(n-1)x\} = 0$

$\Rightarrow \sin(n + 1 - n)x - a\sin(n + 1 - n + 1)x$

$+ a^2\sin(n - n + 1)x = 0$

$\Rightarrow \sin x - a\sin 2x + a^2\sin x = 0$

$\Rightarrow \sin x(a^2 - 2a\cos x + 1) = 0$

$\Rightarrow \sin x[(a - \cos x)^2 + \sin^2 x] = 0$

$\Rightarrow \sin x = 0$

85. (c) $3x^3 + 3x^2y - 3xy^2 + my^3 = 0$

$\Rightarrow my^3 - 3xy^2 + 3x^2y + 3x^3 = 0$

$$\Rightarrow m\left(\frac{y}{x}\right)^3 - 3\left(\frac{y}{x}\right)^2 + 3\left(\frac{y}{x}\right) + 3 = 0 \qquad \ldots\text{(i)}$$

As two lines are at right angles, so roots of Eq. (i) in (y/x) will be $\alpha, -\dfrac{1}{\alpha}$ and β.

$\therefore$ Product of roots

$$\Rightarrow \alpha\left(-\frac{1}{\alpha}\right)\beta = \frac{-3}{m}$$

$\Rightarrow -\beta = -3/m \quad \Rightarrow \quad \beta = 3/m$

Put $(y/x) = \beta = 3/m$ in Eq. (i), we get

$$m \times \frac{27}{m^3} - 3\left(\frac{9}{m^2}\right) + 3\left(\frac{3}{m}\right) + 3 = 0$$

$$\Rightarrow \frac{9}{m} + 3 = 0 \quad \Rightarrow \quad m = -3$$

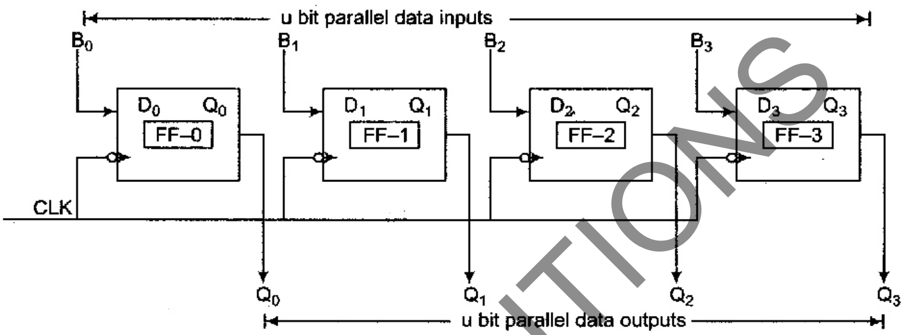
86. (c) The upper triangular matrix

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \\ \vdots \\ \vdots \\ n-1 \end{array}\begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & \dots & & n-1 \end{array} \\ \begin{bmatrix} x & y & z & \dots & \dots & t \\ 0 & a & b & \dots & \dots & c \\ 0 & 0 & \dots & \dots & \dots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & z \end{bmatrix} \end{array}$$

The size of upper triangular matrix
$a[0 \dots\dots n-1, 0, \dots\dots n-1]$
and $n = 50$
$a = [0, \dots\dots 49, 0, \dots\dots, 49]$
Total elements in array $a = 50 \times 50$
$= 2500$
And total elements in array $b = 1275$
So, element of $a[30,40]$ stored in $b[1075]$.

87. (a) Structures can be compared using == operator.

88. (c) Parallel in parallel out shift registers will result in fast data transmission. In this, only one clock pulse is essential to load all the bits.



89. (a) Boolean function

$F(x, y, z) = \Sigma\,(0, 2, 3, 4, 5, 6)$

Draw the k-map for given boolean function

$x \backslash yz$	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{x}$	1 (0)	(1)	1 (3)	1 (2)
x	1 (4)	1 (5)	(7)	1 (6)

Here, one quad and two pairs is made.
Quad is $\rightarrow \bar{y}\,\bar{z} + y\,\bar{z} \rightarrow \bar{z}\,(y + \bar{y}) \rightarrow \bar{z}$
One pair is $\rightarrow x\bar{y}$
Second pair is $\rightarrow \bar{x}y$
$\therefore \quad f = x\bar{y} + \bar{x}y + \bar{z}$

90. (d) $X = BCD$ and the functional dependency

$F = \{A \rightarrow BC, CD \rightarrow E, E \rightarrow C, D \rightarrow AEH,$
$ABH \rightarrow BD, DH \rightarrow BC\}$

We have to calculate X^+ under F

$X^+ = \{BCD\}^+ = \{BCDEAH\}$
$= \{ABCDEH\}$

When we calculate the closure of any attribute, then first of all copy all attribute for which we are deriving closure. Here, we are calculating closure for $\{BCD\}$ attributes, then copy BCD as it is in closure, then check dependency set and find the match.

91. (c) Waiting time of person will lie in the range (0, 30) and in this range favourable portion is (20, 30), so the required probability is the ratio of the time interval's length.

i.e., $\dfrac{10}{30} = \dfrac{1}{3}$

92. (a) $E(e^{tx}) = \int_{-a}^{a} e^{tx} f(x)\, dx$

$$= \int_{-a}^{a} \frac{1}{2a} e^{tx} dx = \left[\frac{e^{tx}}{2at}\right]_{-a}^{a}$$

$$= \frac{e^{at} - e^{-at}}{2at} = \sinh(at)/at$$

93. (a) $\lim_{x \to 1} \sin^{-1}(\log_3 x/3) = \sin^{-1}\left(\log_3 \dfrac{1}{3}\right)$

$= \sin^{-1}(-\log_3 3) = \sin^{-1}(-1) = -\pi/2$

94. (b) Volume of cube = (edge)^3

$\Rightarrow \quad V = E^3$

$\Rightarrow \quad \ln V = \ln E^3 = 3 \ln E$

$\Rightarrow \quad \dfrac{1}{V}\delta V = \dfrac{3}{E}\delta E$

$\Rightarrow \quad \dfrac{\delta V}{V} \times 100\% = 3\dfrac{\delta E}{E} \times 100\% = 3k\%$

95. (a) $(x - y)^2 \dfrac{dy}{dx} = a^2$ ...(i)

Put $x - y = v$

$\Rightarrow \quad 1 - \dfrac{dy}{dx} = \dfrac{dv}{dx}$

$\Rightarrow$ Eq. (i) is

$$v^2\left(1 - \frac{dv}{dx}\right) = a^2$$

$\Rightarrow \quad 1 - \dfrac{dv}{dx} = \dfrac{a^2}{v^2}$

$\Rightarrow \quad \dfrac{dv}{dx} = \dfrac{v^2 - a^2}{v^2}$

$\Rightarrow \quad \dfrac{v^2}{v^2 - a^2} dv = dx$

$\Rightarrow \quad \left(1 + \dfrac{a^2}{v^2 - a^2}\right) dv = dx$

$\Rightarrow \quad v^2 + \dfrac{a^2}{2a} \log\left|\dfrac{v - a}{v + a}\right| = x + K$

$\Rightarrow \quad x - y + \dfrac{a}{2} \log\left|\dfrac{x - y - a}{x - y + a}\right| = x + K$

$\Rightarrow \quad y = \dfrac{a}{2} \log\left|\dfrac{x - y - a}{x - y + a}\right| + C$

96. (a) $(\mathbf{a}+\mathbf{b})\times(\mathbf{a}\times\mathbf{b})$

$= \{(\mathbf{a}+\mathbf{b})\cdot\mathbf{b}\}\mathbf{a} - \{(\mathbf{a}+\mathbf{b})\cdot\mathbf{a}\}\mathbf{b}$

$= (\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{b})\mathbf{a} - (\mathbf{a}\cdot\mathbf{a}+\mathbf{b}\cdot\mathbf{a})\mathbf{b}$

$= (\mathbf{a}\cdot\mathbf{b}+1)\mathbf{a} - (1+\mathbf{a}\cdot\mathbf{b})\mathbf{b}$ $\quad [\because |\mathbf{a}| = |\mathbf{b}| = 1]$

$= (\mathbf{a}-\mathbf{b})(\mathbf{a}\cdot\mathbf{b}+1)$

= Scalar multiple of $\mathbf{a}-\mathbf{b}$

$\Rightarrow$ $(\mathbf{a}+\mathbf{b})\times(\mathbf{a}\times\mathbf{b})$ is parallel to $\mathbf{a}-\mathbf{b}$

97. (d) $\tan\frac{A}{2} = \frac{\Delta}{s(s-a)} = \frac{5}{6}$

$\tan\frac{B}{2} = \frac{\Delta}{s(s-b)} = \frac{20}{37}$

$\Rightarrow \tan\frac{A}{2}\tan\frac{B}{2} = \frac{\Delta^2}{s^2(s-a)(s-b)}$

$\Rightarrow \frac{5}{6}\times\frac{20}{37} = \frac{s(s-a)(s-b)(s-c)}{s^2(s-a)(s-b)}$

$\Rightarrow \frac{s-c}{s} = \frac{50}{111}$

$\Rightarrow \frac{2s-2c}{2s} = \frac{50}{111}$

$\Rightarrow \frac{a+b-c}{a+b+c} = \frac{50}{111}$

$\Rightarrow 111(a+b) - 111c = 50(a+b) + 50c$

$\Rightarrow 61(a+b) = 161c$

$\Rightarrow a+b = \frac{161}{61}c$

98. (b) The equation of circle through points of intersection of $x^2+y^2+13x-3y=0$ and $2x^2+2y^2+4x-7y-25=0$ is

$2x^2+2y^2+4x-7y-25+\lambda(x^2+y^2+13x-3y)=0$ ...(i)

It passes through (1,1), if

$2+2+4-7-25+\lambda(1+1+13-3)=0$

$\Rightarrow \lambda = 2$

put $\lambda = 2$ in Eq.(i), to get the required circle which is

$4x^2+4y^2+30x-13y-25=0$

99. (c) String s_1, s_2;

```
if (strcmp (s1, s2))
printf("strings are equal");
```

This code doesn't work because s_1 and s_2 do not have any value, then how they can be compared with function strcmp.

100. (a) We know that;

N_h = The minimum number of nodes in an AVL tree of height h

$N_{nh} = N_{n-1} + N_{n-2} + 1$

So, at height 1,

$N_1 = N_0 + N_{-1} + 1 = 0+0+1$

$N_1 = 1$

At height 2,

$N_2 = N_1 + N_0 + 1 = 1+0+1$

$N_2 = 2$

At height 3,

$N_3 = N_2 + N_1 + 1 = 2+1+1$

$N_3 = 4$

At height 4,

$N_4 = N_3 + N_2 + 1 = 4+3+1$

$N_4 = 7$

At height 5,

$N_5 = N_4 + N_3 + 1 = 7+4+1$

$N_5 = 12$

At height 6,

$N_6 = N_5 + N_4 + 1 = 12+7+1$

$N_6 = 20$

So, height 6, there are 20 nodes total in AVL tree.

101. (a) The compiler translates the source code into object code and the extension of the object code is .obj.

102. (a) Given that, $A+B > C+D$

$A+C = B+D$

$A = \frac{1}{2}(B+D) \Rightarrow C = \frac{1}{2}(B+D)$

$\Rightarrow A = C$ and $B > D \Rightarrow B > A = C > D$

103. (c) Eigen values of $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 3 \end{bmatrix}$ are λ, then

Characteristic equation $= |A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 3-\lambda & 2 & 2 \\ 2 & 5-\lambda & 2 \\ 2 & 2 & 3-\lambda \end{vmatrix} = 0$

$\Rightarrow 3-\lambda[\lambda^2 - 8\lambda + 11] - 2[2-2\lambda] + 2[2\lambda - 6]$

$\Rightarrow -\lambda^3 + 11\lambda^2 - 27\lambda + 17 = 0$

$\Rightarrow \lambda^3 + 11\lambda^2 + 27\lambda - 17 = 0$

$\Rightarrow (\lambda-1)(\lambda^2 - 10\lambda + 17) = 0$

$\Rightarrow \lambda = 1, \frac{10 \pm \sqrt{32}}{2} = 1, 5 \pm 2\sqrt{2}$

which are contained in (1, 8).

104. (d) Announcement time = 18 : 00 h − 02 : 30 h + 0 : 40 h

= 16:10 h

105. (a) Let $f'(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n$

$\Rightarrow f(x) = \left[a_0\frac{x^{n+1}}{n+1} + a_1\frac{x^n}{n} + \ldots + a_nx\right] + C$

Now, $f(0) = C$

and $f(1) = \left[\frac{a_0}{n+1} + \frac{a_1}{n} + \ldots + a_n\right] + C$

$\left[\because \frac{a_0}{n+1} + \frac{a_1}{n} + \ldots + a_n = 0 \text{ (given)}\right]$

$f(1) = C$

$\Rightarrow f(0) = f(1)$

$\Rightarrow$ By Rolle's theorem, atleast one zero of $f'(x) = 0$ lies in (0, 1).

106. (·) Let $I = \int xe^{x^2}\cos(e^{x^2})\,dx$

Put $y = e^{x^2}$

$\Rightarrow dy = 2xe^{x^2}dx$

$\Rightarrow I = \int \frac{1}{2}\cos y\,dy = \frac{1}{2}\sin y + C = \frac{1}{2}\sin(e^{x^2}) + C$

107. (c) Part of workers having children

$= \frac{1}{3}\times\frac{1}{2}\times\frac{1}{3} + \frac{2}{3}\times\frac{3}{4}\times\frac{2}{3} = \frac{1}{18} + \frac{1}{3} = \frac{7}{18}$

So, the part of worker without children

$= 1 - \frac{7}{18} = \frac{11}{18}$

108. (c) $\frac{\tan\alpha - i(\sin(\alpha/2) + \cos(\alpha/2))}{1 + 2i\sin(\alpha/2)}$

$= \frac{\tan\alpha - i(\sin(\alpha/2) + \cos(\alpha/2))}{1 + 2i\sin(\alpha/2)} \times \frac{1 - 2i\sin(\alpha/2)}{1 - 2i(\sin(\alpha/2))}$

which is purely imaginary, so real part will be zero.

$\Rightarrow \tan\alpha + 2\sin(\alpha/2)[\sin(\alpha/2) + \cos(\alpha/2)] = 0$

$\Rightarrow \tan\alpha + \sin\alpha + 2\sin^2(\alpha/2) = 0$

$\Rightarrow \tan\alpha + \sin\alpha + 1 - \cos\alpha = 0$

$\Rightarrow \alpha = 2n\pi$ because at $\alpha = 2n\pi$, $\tan\alpha$ does not exist.

109. (c) If α and β are roots of $ax^2 + bx + c = 0$, then $\alpha + \beta = -b/a$, $\alpha\beta = c/a$

$$\Rightarrow \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{c^2/a^2}$$

$$= \frac{b^2 - 2ac}{c^2}$$

Now, $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$ (given)

$$\Rightarrow -\frac{b}{a} = \frac{b^2 - 2ac}{c^2}$$

$$\Rightarrow -bc^2 = ab^2 - 2a^2c$$

$$\Rightarrow bc^2 + ab^2 = 2a^2c$$

$$\Rightarrow \frac{c}{a} + \frac{b}{c} = 2\frac{a}{b} \quad \text{(dividing by } abc\text{)}$$

$\Rightarrow \frac{c}{a}, \frac{a}{b}, \frac{b}{c}$ are in AP.

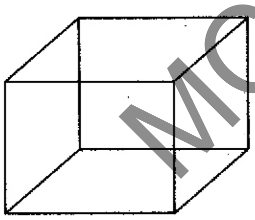
$\Rightarrow \frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in HP.

110. (d) $y = x + 5$ does not touch the circle $x^2 + y^2 = 25$
(By condition of tangency $c^2 \neq a^2(1 + m^2)$
Here, $c = 5, a = 5, m = 1$)

111. (d) int x = 0, int *i = & x; int *j = & x; so the value of i and j are always equal.

112. (a) I. All children are inquisitive.
II. Some children are inquisitive.
III. No children are inquisitive.
IV. Some children are not inquisitive.
Statements I and III cannot be true simultaneously but can both be false.

113. (d) All sides are adjacent to each other, then we require only one colour.



114. (d) In this question, it is not clear, who wear which colour, so we can't say about the colour of Sohan's.

115. (d) $\frac{1}{2!}\left(\frac{h}{b-a}\right)^2 \max|f''(x)| = 1.25 \times 10^{-7}$
which is linear bound of error

$$\Rightarrow \frac{h^2}{8} = 1.25 \times 10^{-7} = \frac{10^{-8}}{8} \quad (\because a = 1, b = 3)$$

$$\Rightarrow h = 10^{-4} \Rightarrow h = 0.0001$$

116. (b) From the given definition

$$\binom{-7.2}{2} = \frac{(-7.2)(-8.2)}{2(1)} = 29.52$$

117. (d) Likelihood function, L is given by

$$L = \prod_{i=1}^{n} (1+\lambda)\, x_i^{\lambda}$$

$$= (1+\lambda)^n (x_1, x_2, \ldots, x_n)^{\lambda}$$

$$\Rightarrow \ln L = n \ln(1+\lambda) + \lambda \Sigma \ln x_i$$

$$\Rightarrow \frac{1}{L}\frac{\partial L}{\partial \lambda} = \frac{n}{1+\lambda} + \Sigma \ln x_i = 0$$

$$\Rightarrow 1 + \lambda = -n/\Sigma \ln x_i$$

$$\Rightarrow \lambda = -1 - \frac{n}{\sum_{i=1}^{n} \ln x_i}$$

118. (c) $g(x) = e^x - 1 - x$

$$\Rightarrow g'(x) = e^x - 1$$

For min value

$$g'(x) = 0 \Rightarrow x = 0$$

$$g''(x) = e^x \Rightarrow g''(0) = 1 > 0 \text{ (min)}$$

$\Rightarrow x = 0$ is local minima point so, minimum value of $g(x)$ is $g(0) = 0$

119. (a) By Chebyshev's inequality,

$$P(|X - \mu| \geq K(\sigma) < \frac{1}{K^2}$$

$$\Rightarrow P(|X - 0.5| \geq K(0.1)) < \frac{1}{K^2}$$

$$\Rightarrow (|X - 0.5| \geq 0.25) < \frac{1}{(5/2)^2} = \frac{4}{25} = 0.16$$

120. (c) Moment generating function of normal variate X with μ and σ^2 is

$$E(e^{tX}) = e^{\mu t + \frac{\sigma^2 t^2}{2}}$$

By putting $t = 1$, we get

$$E(e^X) = e^{\mu + \sigma^2/2}$$