# Solved Paper 2013

# JNU MCA

## Mathematics

**1.** For $a, b \in R$, define $A = \begin{bmatrix} 1 & a \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ a & b \end{bmatrix}$

**Statement I** For any $a$, $A$ is diagonalisable matrix.

**Statement II** For any $a$ and $b$, 1, $B$ is a diagonalisable matrix.

(a) Statement I is true, Statement II is false
(b) Statement I is false, Statement II is true
(c) Both statements are false
(d) Both statements are true

**2.** If @ means triple of, # means double of and ^ means half of, then the value of @ # ^ @ ^ 5 + @ # @ ^ 2 is

(a) 39.5 (b) 40.5
(c) 39.74 (d) None of these

**3.** A ray of light incident at the point $(-2, -1)$ gets reflected from the tangent at $(0, -1)$ to the circle $x^2 + y^2 = 1$. This refracted ray touches the circle. The equation of the line along which the incident ray moved, is

(a) $4x - 3y + 11 = 0$ (b) $4x + 3y + 11 = 0$
(c) $3x + 4y + 11 = 0$ (d) None of these

**4.** What is the next term in the sequence 49, 121, 225, 361,...?

(a) 400 (b) 441
(c) 481 (d) 529

**5.** The number of pairs $(a, b)$ for which $a(x+1)^2 - b(x^2 - 3x + 2) + x + 1 = 0, \forall x \in R$, is

(a) 0 (b) 1
(c) 2 (d) infinite

**6.** If a function $y = f(x)$ is defined parametrically as $y = t^2 + t|t|, x = 2t - |t|, t \in R$. Then, at $x = 0$, $f(x)$ is

(a) continuous but non-differentiable
(b) differentiable
(c) discontinuous
(d) None of the above

**7.** The value of $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3)$ is equal to

(a) $\pi$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) None of these

**8.** The number of positive integral solutions of $\tan^{-1}(x) + \cot^{-1}(y) = \tan^{-1}(3)$ is

(a) one (b) two (c) three (d) four

**9.** The interval in which $\cos^{-1}(x) > \sin^{-1}(x)$, is

(a) $(-\infty, 1)$ (b) $(-1, 1)$
(c) $(-1, 1/\sqrt{2})$ (d) $[-1, 1]$

**10.** If $(1, -1, -1)^T$ is an eigen vector of the matrix

$$\begin{bmatrix} 4 & 1 & 1 \\ -5 & 0 & -3 \\ -1 & -1 & 2 \end{bmatrix}$$

Then, the corresponding eigen value is

(a) 2 (b) $-2$ (c) $-1$ (d) 3

**11.** A father's age is 4 times the age of his elder son and 5 times the age of his younger son. When the elder son lived to three times his present age, then the father's age will exceed his younger son's age by 3 yr. What is the age of the father?

(a) 40 yr (b) 32 yr
(c) 30 yr (d) None of these

**12.** The perimeter of $\Delta ABC$ is 6 times the arithmetic mean of the sine of its angles. If side $a$ is 1, then $\angle A$ is

(a) 30° (b) 60°
(c) 90° (d) 120°

**13.** Out of the 18 points in a plane, no three points are in the straight line except 5 points which are collinear. The number of straight lines that can be formed joining them, is

(a) 155 (b) 153 (c) 144 (d) 143

**14.** The orthocentre of the triangle formed by the lines $x + y = 1, 2x + 3y = 6$ and $4x - y + 4 = 0$ lies in

(a) I quadrant (b) II quadrant
(c) III quadrant (d) IV quadrant

**15.** If the system of straight lines $a(2x + y - 3) + b(3x + 2y - 5) = 0$, then the line of the system situated farthest from the point $(4, -3)$ has the equation

(a) $4x + 11y - 15 = 0$ (b) $7x + y - 8 = 0$
(c) $4x + 3y - 7 = 0$ (d) $3x - 4y + 1 = 0$

**16.** If $A + B = \frac{\pi}{3}$, where $A,B > 0$, then the minimum value of $\sec A + \sec B$ is equal to

(a) $\frac{4}{\sqrt{3}}$ (b) $\frac{8}{\sqrt{3}}$
(c) 6 (d) None of these

**17.** If $a$, $b$, $c$ and $d$ are distinct real numbers such that $(a^2 + b^2 + c^2)x^2 - 2x(ab + bc + cd) + b^2 + c^2 + d^2 \le 0$, then they satisfy

(a) AP (b) GP
(c) HP (d) $ab = cd$

**18.** The equation $y^2 - x^2 + 2x - 1 = 0$ represents

(a) a pair of straight lines (b) a circle
(c) a parabola (d) an ellipse

**19.** The dimension of a subspace of $R^4$ spanned by the vectors $\{(2, -1, 0, 1), (1, 2, -3, 2), (1, -3, 2, 0), (0, 0, 1, -1)\}$ is

(a) 1 (b) 2
(c) 3 (d) 4

**20.** In what ratio should wheat at ₹ 4.5 per kg be mixed with another variety at ₹ 5.25 per kg, so that the mixture is worth of ₹ 5 per kg?

(a) 1 : 2 (b) 2 : 1
(c) 1 : 3 (d) 3 : 1

**21.** The value of $p$, for which the roots of the equation $(p-3)x^2 - 2px + 5p = 0$ are real and positive, are

(a) $p \in (3, 15/4]$ (b) $p \in (3, 15/4)$
(c) $p \in [3, 15/4)$ (d) $p \in [3, 15/4]$

**22.** The value of $\lim_{x \to \infty} \frac{(1 + x + x^2)}{x(\ln x)^3}$ is equal to

(a) 2 (b) $e^2$
(c) $e^{-2}$ (d) None of these

**23.** The value of
$2\sin^2\theta + 4\cos(\theta + \phi)\sin\alpha\sin\theta + \cos(2\alpha + 2\theta)$ is

(a) $\cos\theta + \cos\alpha$
(b) independent of $\theta$
(c) independent of $\alpha$
(d) None of the above

**24.** Which of the following is rational number?

(a) sin15° (b) cos15°
(c) sin15° cos15° (d) sin15° cos75°

**25.** The value of $\int_C \left(\frac{z+1}{z-1}\right)^3 dz$ around a circular contour $C = \{z : |z| < 2\}$, is

(a) 0 (b) $3\pi i$
(c) $6\pi i$ (d) $12\pi i$

**26.** Two taps can fill a tank in 18 min and 24 min, respectively. When both the taps are opened, find when the first tap is turned off so that the tank may be filled in 12 min.

(a) After 6 min (b) After 10 min
(c) After 9 min (d) After 12 min

**27.** If $\cos x / a = \sin x / b$, then $|a\cos 2x + b\sin 2x|$ is

(a) $\sqrt{a^3 b}$ (b) $a^2/|b|$ (c) $b^2/|a|$ (d) $|a|$

**28.** The area of the triangle formed by the lines $y = ax$, $x + y - a = 0$ and the $Y$-axis, is equal to

(a) $1/2|1 + a|$ (b) $a^2/|1 + a|$
(c) $1/2|1/(1 + a)|$ (d) $a^2/2|1 + a|$

**29.** A variable point $[1 + \alpha/\sqrt{2}, 2 + \alpha/\sqrt{2}]$, lies in between two parallel lines $x + 2y = 1$ and $2x + 4y = 15$. Then, the range of $\alpha$ is given by

(a) $0 < \alpha < 5\sqrt{2}/6$ (b) $-4\sqrt{2}/3 < \alpha < 5\sqrt{2}/6$
(c) $-4\sqrt{2}/3 < \alpha < 0$ (d) None of these

**30.** $A$ and $B$ are two fixed points. The vertex $C$ of $\Delta ABC$ moves such that $\cot A + \cot B$ = constant. The locus of $C$ is a straight line

(a) perpendicular to $AB$
(b) parallel to $AB$
(c) inclined at an angle $(A - B)$ to $AB$
(d) None of the above

**31.** A straight line $L$ with negative slope passes through the point (8, 2) and cuts the positive coordinate axes at points $P$ and $Q$. As $L$ varies, the absolute minimum value of $(OP + OQ)$, where $O$ is the origin, is

(a) 10 (b) 18 (c) 16 (d) 12

**32.** If a circle passes through the points of intersection of the lines $2x - y + 1 = 0$ and $x + ay - 3 = 0$ with the axes of the reference, then the value of $a$ is

(a) 0.5 (b) 2 (c) 1 (d) $-2$

**33.** A foot of the normal from the point (4, 3) to a circle is (2, 1) and the diameter of the circle has the equation $2x - y = 2$. Then, the equation of the circle is

(a) $x^2 + y^2 + 2x - 1 = 0$ (b) $x^2 + y^2 - 2x - 1 = 0$
(c) $x^2 + y^2 + 2y - 1 = 0$ (d) None of these

**34.** The radius of convergence of the power series

$\sum_{n=0}^{\infty} \frac{(n!)^3}{3n!} z^n$ is

(a) 1/27 (b) 27
(c) 1/3 (d) 3

**35.** The average minimum temperature for Monday, Tuesday and Wednesday was 4°C and that for Tuesday, Wednesday and Thursday was 5.5°C. If the temperature on Monday was 2.6°C, then what was the temperature on Thursday?

(a) 4.1°C (b) 12.1°C
(c) 7.1°C (d) 11 1°C

**36.** The equation of the circle touching the line $|y| = x$ at a distance $\sqrt{2}$ units from the origin, is

(a) $x^2 + y^2 - 4x + 2 = 0$ (b) $x^2 + y^2 + 4x - 2 = 0$
(c) $x^2 + y^2 + 4x + 2 = 0$ (d) None of these

## JNU MCA Solved Paper 2013

**37.** Circles with radii 3, 4 and 5 touch each other externally. $P$ is the point of intersection of tangents to these circles at their points of contact. The distance of $P$ from the point of contact is

(a) $\sqrt{3}$ (b) $\sqrt{4}$
(c) $\sqrt{5}$ (d) 12

**38.** The angle between the circles

$C_1 \equiv x^2 + y^2 - 4x + 6y + 11 = 0$

$C_2 \equiv x^2 + y^2 - 2x + 8y + 13 = 0$

is equal to

(a) 15° (b) 30°
(c) 45° (d) 60°

**39.** For $1 < |z| < 4$, the coefficient of $z^2$ in the Laurent series expansion of $\frac{1}{z^2 - 5z + 4}$ is equal to

(a) 1/192 (b) 1/48
(c) –1/48 (d) –1/192

**40.** A sum of ₹ 3310 is to be paid back in 3 equal annual instalments. What is the total interest charged, if the interest is compounded annually at 10%?

(a) ₹ 1331 (b) ₹ 683
(c) ₹ 331 (d) ₹ 993

**41.** The greatest integer which divides the number $101^{100} - 1$, is equal to

(a) 100 (b) 1000
(c) 10000 (d) None of these

**42.** The sum of the coefficients of all the integral powers of $x$ in the expansion of $(1 + 2\sqrt{x})^{40}$, is

(a) $3^{40} + 1$ (b) $3^{40} - 1$
(c) $1/2(3^{40} - 1)$ (d) $1/2(3^{40} + 1)$

**43.** If $(1 + x)^n = C_0 + C_1 x + \dots + C_n x^n$, then the value of

$\sum_{r=0}^{n} \sum_{z=0}^{n} (C_r + C_s)$ is

(a) $(n + 1)2^{n+1}$ (b) $(n - 1)2^{n+1}$
(c) $(n + 1)2^n$ (d) None of these

**44.** Which of the following is not the property of ${}^nC_r$?

(a) ${}^nC_1 = {}^nC_{n-1}$
(b) ${}^nC_r = {}^nC_{n-r}$
(c) $r\,{}^nC_r = n\,{}^{n-1}C_{r-1}$
(d) $(r - 1)\,{}^nC_r = (n - 1)\,{}^{n-1}C_{r-1}$

**45.** A city has 12 gates. In how many ways can a person enter the city through one gate and come out through a different gate?

(a) 144
(b) 132
(c) 12
(d) None of the above

**46.** A bag contains 5 paisa coins, 10 paisa coins and 20 paisa coins in the ratio of 3 : 2 : 1. If their total value is ₹ 11, then what is the number of 5 paisa coins?

(a) 50 (b) 60
(c) 120 (d) 200

**47.** In how many ways can 5 letters be posted in 4 letter boxes?

(a) 20 (b) 32
(c) $4^5$ (d) $5^4$

**48.** If the length of the sides of a triangle are 3, 4 and 5 units, then the circumradius is

(a) 2.0 units (b) 2.5 units
(c) 3.0 units (d) 3.5 units

**49.** The sides of a triangle are 17, 25 and 28. The greatest altitude of length is

(a) 15 (b) 84/5
(c) 420/17 (d) None of these

**50.** The straight line $y = mx + c$ touches the parabola $y^2 = 4a(x + a)$, if

(a) $c = am - a/m$ (b) $c = m - a/m$
(c) $c = am + a/m$ (d) None of these

**51.** $P$ is any point on the ellipse $81x^2 + 144y^2 = 1944$, whose foci are $S$ and $S'$. Then, $SP + S'P$ is equal to

(a) 3 (b) $4\sqrt{6}$ (c) 36 (d) 324

**52.** The locus of a variable point, whose distance from $(-2, 0)$ is 2/3 times its distance from the line $x = -9/2$, is

(a) ellipse (b) parabola
(c) hyperbola (d) None of these

**53.** If $0 < a < b$, then $\lim_{n \to \infty} (b^n + a^n)^{1/n}$ is equal to

(a) e (b) $a$
(c) $b$ (d) None of these

**54.** The value of $\lim_{x \to 0} [\sin x/x]$, where [ ] denotes the greatest integer function

(a) is equal to 1 (b) is equal to zero
(c) does not exist (d) None of these

**55.** $\lim_{n \to \infty} \int_0^1 \frac{nx^{n-1}}{1 + x} dx$

(a) does not exist (b) is equal to 1
(c) is equal to 0 (d) is equal to 0.5

**56.** A shopkeeper marks his goods 20% above his cost price but allows a discount of 8% for cash. What is his profit per cent?

(a) 10.6 (b) 11.6 (c) 9.6 (d) 10.4

**57.** If $A + B + C = \pi$, then the greatest value of $\cos A + \cos B + \cos C$ is

(a) 2 (b) 3
(c) 3/2 (d) 1

**58.** If $f, g : [0, 1] \to R$ are defined as

$$f(x) = \begin{cases} 1, & x = 0, \frac{1}{n!}, \frac{2}{n!}, \dots, \frac{n}{n!}, n \in N \\ 0, & \text{otherwise} \end{cases}$$

and $g(x) = \begin{cases} 1, & x = \frac{1}{n}, n \in N \\ 0, & \text{otherwise} \end{cases}$

Then,

(a) both $f$ and $g$ are Riemann integrable on [0, 1]
(b) $f$ is Riemann integrable on [0, 1] but $g$ is not
(c) $g$ is Riemann integrable on [0, 1] but $f$ is not
(d) both $f$ and $g$ are not Riemann integrable on [0, 1]

**59.** At what time between 6 and 7 do the hands of the clock coincide?

(a) 6:13 (b) 6:36
(c) 6:32 (d) 6:35

**60.** A lamp of negligible height is placed on the ground $t$ metre away from a wall. A man $m$ metre tall is walking at a speed of 0.1 $t$ m/s from the lamp to the nearest point on the wall. When he is midway between the lamp and the wall, the rate of change in the length of his shadow on the wall is

(a) $-2.5t$ m/s (b) $-0.4\,m$ m/s
(c) $-0.2\,t$ m/s (d) $-0.2\,m$ m/s

**61.** The total number of critical points of $f(x) = \max(\sin x, \cos x)$, $\forall\, x \in (-2\pi, 2\pi)$, is equal to

(a) 3 (b) 4 (c) 5 (d) 7

**62.** A group of 7 students with average weight of 66 kg is joined by another 5 students with average weight of 64 kg. What is the average weight of the total group?

(a) 64.8 kg (b) 64.5 kg (c) 65.3 kg (d) 65.17 kg

**63.** If $P(x)$ is a non-constant cubic polynomial with real coefficients and $Q(x) = P(x) - \sqrt{2}P'(x)$. Then, $Q(x) = 0$ has

(a) all roots real (b) all roots complex
(c) exactly one real root (d) exactly one complex root

**64.** A rectangular tank is 4 m long, 3 m wide and 1.5 m high and is filled with water upto 0.6 m. How many stones of 15 cm × 10 cm × 8 cm are to be dropped to take the water to the top of the tank?

(a) 9000 (b) 10000
(c) 1000 (d) 900

**65.** If $f(x) = 2x - \tan^{-1} x - \ln(x + \sqrt{1 + x^2})$, $\forall\, x \in R$, then

(a) $f(x)$ is non-increasing in $(-\infty, \infty)$
(b) $f(x)$ is non-decreasing in $(-\infty, \infty)$
(c) $f(x)$ is increasing in $(-\infty, \infty)$
(d) $f(x)$ is decreasing in $(-\infty, \infty)$

**66.** If $\alpha, \beta$ are the roots of $x^2 - x - 1 = 0$ and $A_n = \alpha^n + \beta^n$, then the arithmetic mean of $A_n$ and $A_{n-1}$ is

(a) $2A_{n+1}$ (b) $\frac{1}{2}A_{n+1}$
(c) $2A_{n-2}$ (d) None of these

**67.** The slope of the normal at the point with abscissa $x = -2$ of the graph of the function $f(x) = |x^2 - x|$, is

(a) $-\frac{1}{6}$ (b) $-\frac{1}{3}$ (c) $\frac{1}{6}$ (d) None of these

**68.** 10.9876543210 is divisible by

(a) 5, 9 and 11 (b) 5 and 9 but not by 11
(c) 9 and 11 but not by 5 (d) 11 and 5 but not by 9

**69.** What is the digit in the unit's place of the product $23^{49} \times 51^{36}$?

(a) 1 (b) 3 (c) 7 (d) 9

**70.** If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + bx + c$, where $ac \neq 0$, then the equation $P(x)\,Q(x) = 0$ has

(a) one real root (b) two real roots
(c) atleast two real roots (d) atmost two real roots

**71.** If $\{a_n\}$ is a real sequence such that $a_0 > 0, a_1 > 0$ and $a_n = \sqrt{a_{n-1}} + \sqrt{a_{n-2}}$, $n \geq 2$, then $\{a_n\}$

(a) does not converge (b) converges to $\sqrt{2}$
(c) converges to 2 (d) converges to 4

**72.** The population of a town was 24000. If males increase by 6% and females by 9%, it becomes 25620. The number of males and females were

(a) 19000, 5000 (b) 16000, 8000
(c) 20000, 4000 (d) 18000, 6000

**73.** If $\sin^6\theta + \cos^6\theta + k\cos^2 2\theta = 1$, then $k$ is equal to

(a) $1/2 \tan^2 2\theta$ (b) $1/4 \tan^2 2\theta$
(c) $4 \cot^2 2\theta$ (d) $3/4 \tan^2 2\theta$

**74.** A sum of money amounts to ₹ 6050 in 2 yr and ₹ 6655 in 3 yr at compound interest, being compounded annually. Find the sum and the rate of interest.

(a) ₹ 4000, 12% (b) ₹ 5000, 10%
(c) ₹ 5500, 10% (d) None of these

**75.** If today is Tuesday, then what day of the week was it 124 days before?

(a) Wednesday (b) Thursday
(c) Tuesday (d) Friday

**76.** The Fourier series expansion of the function

$$f(x) = \begin{cases} -x, & -4 \leq x \leq 0 \\ x, & 0 \leq x \leq 4 \end{cases}$$

in the interval $[-4, 4]$ involves

(a) no cosine terms
(b) no sine terms
(c) no constant term but both sine and cosine terms
(d) a non-zero constant term but both sine and cosine terms

**77.** Let $(C[0,5], d_\infty)$ be a metric space, where $C[0,5]$ is the set of all real-valued continuous functions on $[0,5]$ and $d_\infty(f, g) = \sup\{|f(x) - g(x)| : x \in [0,5]\}$. If $f(x) = x^2 - 4x$ and $g(x) = 3x - 6$, then $d_\infty(f, g)$ is equal to

(a) 6.0 (b) 6.25
(c) 6.15 (d) 6.35

# JNU MCA Solved Paper 2013

**78.** Three solid cubes of edges 3 cm, 4 cm and 5 cm are melted to form a new cube. The edge of the new cube is

(a) 7.7 cm (b) 6.0 cm (c) 6.5 cm (d) 8.0 cm

**79.** If 'BATCH' is coded as 'ABSDG', then what is the code for 'TERM'?

(a) PFSL (b) SFQL
(c) SFQN (d) None of these

**80.** If $g(a+b-x) = g(x)$, then $\int_a^b x\, g(x)\, dx$ is equal to

(a) $\frac{(a+b)}{2}\int_a^b g(b-x)dx$ (b) $\frac{(a+b)}{2}\int_a^b g(b+x)dx$
(c) $\frac{(b-a)}{2}\int_a^b g(x)dx$ (d) $\frac{(a+b)}{2}\int_a^b g(x)dx$

**81.** Suppose all five men at a party throw their hats in the centre of the room. Each man, then randomly selects a hat. The probability that none of the five men selects his own hat is

(a) $\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}$ (b) $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$
(c) $1-\frac{1}{2!}+\frac{1}{3!}-\frac{1}{4!}+\frac{1}{5!}$ (d) None of these

**82.** A particular solution of the differential equation $(x-y)(dx+dy) = dx - dy$, given that $y = -1$, when $x = 0$, is

(a) $\log|x+y| = x-y+1$ (b) $\log|x+y| = x+y+1$
(c) $\log|x^2+y^2| = x-y+1$ (d) None of these

**83.** The planes $2x-3y+4z-5=0$ and $5x-2.5y+10z-6=0$

(a) are perpendicular (b) are parallel
(c) intersect Y-axis (d) pass through (0, 0, 1.25)

**84.** If $n \in N$, the set of natural numbers. Define a sequence of functions on $(-1, 1)$ as

$$f_n(x) = \begin{cases} \frac{2-(1+x)^n}{n}, & -1 < x < 0 \\ \frac{(1-x)^n}{n}, & 0 < x < 1 \end{cases}$$

Then, $\{f_n(x)\}$

(a) does not converge on (– 1, 1)
(b) converges only for $x = 0$
(c) converges on (– 1, 1)
(d) converges on (– 1, 1), except for $x = 0$

**85.** A number is multiplied by 9 and 9 is added to it. If the result is divisible by 17, then the number is

(a) 12 (b) 15 (c) 17 (d) 16

**86.** The cumulative distribution function of a random variable is

$$f(x) = \begin{cases} 0, & x \le \alpha \\ \frac{x-\alpha}{\beta-\alpha}, & \alpha < x < \beta \\ 1, & x \ge \beta \end{cases}$$

The random variate follows which of the following probability distributions?

(a) Uniform
(b) Exponential
(c) Binomial
(d) Poisson

**87.** The maximum value of $xyz$, when $x^2+y^2+z^2=1$, is

(a) $\frac{1}{3\sqrt{3}}$ (b) 1
(c) $\frac{1}{27}$ (d) None of these

**88.** The value of $\int \frac{dx}{\sqrt{\sin^3 x \sin(x+\alpha)}}$ is

(a) $\frac{2}{\sin\alpha}\sqrt{\frac{\sin(x+\alpha)}{\sin x}}+C$ (b) $-\frac{1}{2\sin\alpha}\sqrt{\frac{\sin(x+\alpha)}{\sin x}}+C$
(c) $-\frac{2}{\sin\alpha}\sqrt{\frac{\sin(x+\alpha)}{\sin x}}+C$ (d) None of these

**89.** The value of

$1+\frac{2^3}{2!}+\frac{3^3}{3!}+\frac{4^3}{4!}+\ldots$ is

(a) $\infty$ (b) $\pi$
(c) $e$ (d) None of these

**90.** If 830 is divided into three terms such that 4 times the first term is equal to 5 times the second and 7 times the third, then the first term is

(a) 350 (b) 280
(c) 200 (d) 230

**91.** In a row of 21 girls, when *M* was shifted by four places towards right, she became 12th from the left end. What was her earlier position from the right end?

(a) 11th (b) 12th
(c) 13th (d) 14th

**92.** A solid cube is painted black on two of its opposite faces and is cut into 343 equal pieces. How many small pieces have no paint?

(a) 245
(b) 313
(c) 294
(d) None of the above

**93.** The area of the circle $x^2+y^2=16$ exterior to the parabola $y^2=6x$ is equal to

(a) $\frac{4(4\pi-\sqrt{3})}{3}$ (b) $\frac{4(4\pi+\sqrt{3})}{3}$
(c) $\frac{4(8\pi-\sqrt{3})}{3}$ (d) $\frac{4(8\pi+\sqrt{3})}{3}$

**94.** The line $y-ax+1=0$ is a tangent to the curve $y^2-4x=0$, if the value of $a$ is

(a) –1 (b) 2
(c) 3 (d) $\frac{1}{2}$

**95.** Let $X, Y$ and $Z$ be three independent normal (0,1) random variables. Calculate $E[\{X+Y+Z\}^2]$.

(a) 0 (b) 1 (c) 3 (d) 9

**96.** A fair coin is tossed 100 times. The probability of getting exactly 50 heads is close to

(a) 0.001 (b) 0.1 (c) 0.5 (d) 0.9

**97.** A number is randomly chosen from the interval (0, 1). What is the probability that its second decimal digit will be 5?

(a) 0.1 (b) 0.5 (c) 0.01 (d) 0.07

**98.** If $f(x) = \frac{4^x}{4^x + 2}$, then find

$$f\left(\frac{1}{2001}\right) + f\left(\frac{2}{2001}\right) + \ldots + f\left(\frac{2000}{2001}\right).$$

(a) 1000 (b) 2000

(c) $\frac{1000}{2001}$ (d) Cannot be determined

**99.** If

$$D_r = \begin{vmatrix} r & x & \frac{n(n+1)}{2} \\ 2r-1 & y & n^2 \\ 3r-2 & z & \frac{n(3n-1)}{2} \end{vmatrix}$$

Compute $\sum_{r=1}^{n} D_r$.

(a) $n$ (b) $\frac{n(n+1)(2n+1)}{6}$

(c) 1 (d) 0

**100.** An equilateral triangle is drawn in a unit square such that one of its vertices coincides with a vertex of the square. What is the maximum possible area of the triangle?

(a) $2\sqrt{3}-3$ (b) $\sqrt{3}-1$ (c) $2\sqrt{3}$ (d) $3\sqrt{3}$

**101.** The amount of bread [in hundreds of kilograms] that a bakery sells in a day is a random variable with density

$$f(x) = \begin{cases} kx, & 0 \le x < 3 \\ k(6-x), & 3 \le x < 6 \\ 0, & 0 \end{cases}$$

Find the value of $k$ which makes $f$ a probability density function.

(a) 3 (b) $\frac{1}{3}$ (c) 9 (d) $\frac{1}{9}$

**102.** The value of $\int \frac{dx}{x\sqrt{ax - x^2}}$ is

(a) $-\left(1+\frac{1}{x^4}\right)^{5/4} + C$ (b) $-\left(1+\frac{1}{x^3}\right)^{1/4} + C$

(c) $\left(1+\frac{1}{x^4}\right)^{1/4} + C$ (d) None of these

**103.** The value of $\int_{\pi/6}^{\pi/3} \frac{(\sin x + \cos x)}{\sqrt{\sin 2x}}\, dx$ is

(a) $\sin^{-1}\frac{(\sqrt{3}-1)}{1}$ (b) $\tan^{-1}\frac{(\sqrt{3}-1)}{1}$

(c) $\sec^{-1}\frac{(\sqrt{3}-1)}{1}$ (d) None of these

**104.** Equal quantities of 1 : 5 and 3 : 5 milk to water solutions are mixed together. What will be the ratio of milk to water in the new mixture?

(a) 13 : 35

(b) 2 : 5

(c) 5 : 8

(d) 35 : 13

**105.** In a kilometre race, $P$ beats $Q$ by 25 m or 5s. Find the time taken by $P$ to complete the race.

(a) 3 min 15 s

(b) 4 min 20 s

(c) 2 min 30 s

(d) 5 min 10 s

**106.** The points on the curve $9y^2 - x^3 = 0$, where the normal to the curve makes equal intercepts with the axes, are

(a) $\left(4, \pm\frac{8}{3}\right)$ (b) $\left(4, -\frac{8}{3}\right)$

(c) $\left(4, \pm\frac{3}{8}\right)$ (d) $\left(\pm 4, \frac{3}{8}\right)$

**107.** The truncated Poisson distribution with the zero class missing has probability function

$$P(x=k) = \frac{\lambda^k}{(e^\lambda - 1)k!}, k = 1, 2, \ldots$$

Compute $E[x]$.

(a) $\lambda$ (b) $\frac{1}{\{e^\lambda - 1\}}$

(c) $\frac{\lambda}{\{1-e^{-\lambda}\}}$ (d) $\frac{\lambda^2}{\{1-e^{-\lambda}\}}$

**108.** A number when divided by 238 leaves a remainder of 79. What will be the remainder when the number is divided by 17?

(a) 8 (b) 9 (c) 10 (d) 11

**109.** A balance shows 990 gm for 1 kg. Find the profit of trader, if he marks his goods up by 20% of CP.

(a) 7% (b) 10%

(c) 11% (d) 8%

**110.** Distance between the two planes $2x + 3y + 4z - 4 = 0$ and $4x + 6y + 8z - 12 = 0$ is equal to

(a) 8

(b) 4

(c) 2

(d) None of the above

# Answer *with* Explanations

**1.** (c) Given, $A=\begin{bmatrix}1 & a\\0 & 2\end{bmatrix}$

$$B=\begin{bmatrix}1 & 0\\a & b\end{bmatrix}$$

Both $A$ and $B$ wil be diagonalisable, when $a=0$ and $b$ can be any real number.

$\therefore$ Both statements are false.

**2.** (b) @ # ^@ ^5+@ # @ ^2

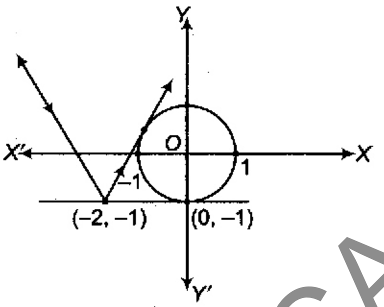
$$=3\left[2\left\{\frac{1}{2}\left(3\left(\frac{1}{2}\times5\right)\right)\right\}\right]+3\left[2\left\{3\left(\frac{1}{2}\times2\right)\right\}\right]$$

$$=\frac{45}{2}+18=\frac{45+36}{2}$$

$$=\frac{81}{2}=40.5$$

**3.** (b) We know that, the equation of tangents of slope $m$ to the circle $x^2+y^2=r^2$ are

$$y=mx\pm r\sqrt{1+m^2} \quad \text{...(i)}$$




Since, tangent (i) passes through the point $(-2,-1)$.

$\therefore \quad -1=-2m\pm\sqrt{1+m^2}$

$\Rightarrow \quad -1+2m=\pm\sqrt{1+m^2}$

$\Rightarrow \quad (2m-1)^2=1+m^2$

$\Rightarrow \quad 4m^2+1-4m=1+m^2$

$\Rightarrow \quad 3m^2-4m=0$

$\Rightarrow \quad m(3m-4)=0$

$\Rightarrow \quad m=0,\frac{4}{3}$

$\therefore$ $m=0$ is the slope of tangent to the circle at $(0,-1)$.

Slope of reflected ray, $m=\frac{4}{3}$

Now, slope of incident ray $=-\frac{4}{3}$

$\therefore$ Equation of line moves along incident ray,

$$(y+1)=-\frac{4}{3}(x+2)$$

$\Rightarrow \quad 4x+3y+11=0$

**4.** (d) Given, 49, 121, 225, 361, ...

$\therefore \quad 7^2, 11^2, 15^2, 19^2, \ldots$

Here, we see that each term after the first term follow a pattern that each term is square of a number, which is 4 more than preceding number without squaring.

$\therefore$ Next term $=(19+4)^2=(23)^2=529$

**5.** (b) $\quad a(x+1)^2-b(x^2-3x+2)+x+1=0$

$\Rightarrow a(x^2+1+2x)-b(x^2-3x+2)+x+1=0$

$\Rightarrow (a-b)x^2+(2a+3b)x+x+a-2b+1=0$

$\Rightarrow (a-b)x^2+(2a+3b+1)x+a-2b+1=0$

$\because \quad x\in R$

$\therefore \quad D\ge0$

$\Rightarrow (2a+3b+1)^2-4(a-b)(a-2b+1)\ge0$

$\Rightarrow 4a^2+9b^2+1+12ab+6b+4a-4(a^2-2ab+a-ab+2b^2-b)\ge0$

$\Rightarrow 4a^2+9b^2+1+12ab+6a+4a-4a^2+8ab-4a+4ab-8b^2+4b\ge0$

$\Rightarrow b^2+24ab+6a+4b\ge0$

$\Rightarrow b^2+b(24a+4)+6a\ge0$

Hence, only one pair of $(a,b)$ exists for which above expression is greater than or equal to zero.

**6.** (b) $y=t^2+t|t|$

$$x=2t-|t|, t\in R$$

$$\therefore \quad \frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{4t}{1}$$

When, $x=0, t=0$

$$\left(\frac{dy}{dx}\right)_{t=0}=0$$

So, $y=f(x)$ is differentiable function.

**7.** (a) $\tan^{-1}(1)+\tan^{-1}(2)+\tan^{-1}(3)$

$$=\pi+\tan^{-1}\left(\frac{1+2}{1-2}\right)+\tan^{-1}3$$

$$=\pi+\tan^{-1}(-3)+\tan^{-1}(3)$$

$$=\pi-\tan^{-1}(3)+\tan^{-1}(3)=\pi$$

**8.** (b) $\tan^{-1}x+\cot^{-1}y=\tan^{-1}(3)$

$\Rightarrow \quad \tan^{-1}x+\tan^{-1}\left(\frac{1}{y}\right)=\tan^{-1}(3)$

$\Rightarrow \quad \tan^{-1}x-\tan^{-1}(3)=\tan^{-1}\left(\frac{1}{y}\right)$

$\Rightarrow \quad \tan^{-1}\left(\frac{x-3}{1+3x}\right)=\tan^{-1}\left(\frac{1}{y}\right)$

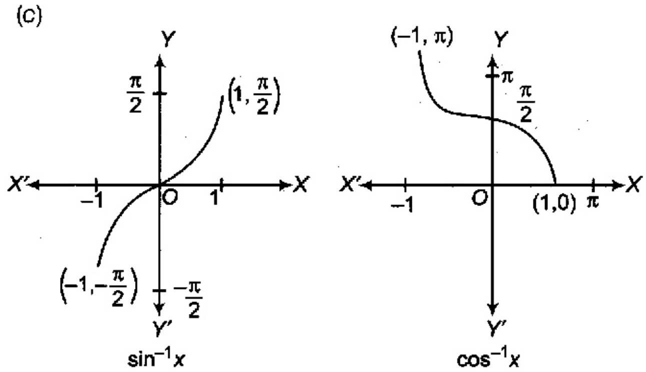
$$\Rightarrow \qquad \frac{x-3}{1+3x} = \frac{1}{y}$$

$$\Rightarrow \qquad y = \frac{1+3x}{x-3}, x > 3$$

It has only positive integral solutions at $x = 5, 8$.

$\therefore$ Total number of solutions are two.

**9.** (c)




It is clear from the graph that

$\cos^{-1} x > \sin^{-1} x$ for the interval $\left(-1, \frac{1}{\sqrt{2}}\right)$.

**10.** (a) $A - \lambda I = 0$, eigen vector $X = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

$$\therefore \begin{bmatrix} 4-\lambda & 1 & 1 \\ -5 & -\lambda & -3 \\ -1 & -1 & 2-\lambda \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4-\lambda-1-1 \\ -5+\lambda+3 \\ -1+1-2+\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \qquad 4 - \lambda - 2 = 0$$

$$\Rightarrow \qquad \lambda = 2$$

**11.** (d) Let father's age be $x$ yr.

Elder son's age $= \frac{1}{4} x$ yr

Younger son's age $= \frac{1}{5} x$ yr

According to the question,

$$\frac{3}{4} x = \frac{1}{4}\left(\frac{14x}{20} + 3\right)$$

$$\Rightarrow \qquad 3x = \frac{14x}{20} + 3$$

$$\Rightarrow \qquad 60x = 14x + 60$$

$$\Rightarrow \qquad 46x = 60$$

$$\therefore \qquad x = \frac{60}{46} \qquad \text{[not possible]}$$

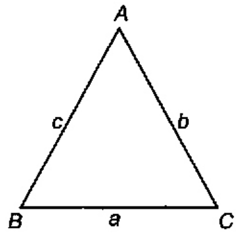
**12.** (a) According to the question,

$$a + b + c = 6 \cdot \frac{\sin A + \sin B + \sin C}{3}$$

$$\Rightarrow a + b + c = 2(\sin A + \sin B + \sin C) \qquad \text{..(i)}$$

By sine rule, we have

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k \qquad \text{...(ii)}$$




$$\Rightarrow \qquad \sin A = ak$$

$$\sin B = bk$$

$$\sin C = ck$$

Substituting in Eq. (i), we get

$$a + b + c = 2k(a + b + c)$$

$$\Rightarrow \qquad k = \frac{1}{2}$$

Now, $\qquad \sin A = \frac{a}{2} = \frac{1}{2} \qquad [\because a = 1]$

$$\therefore \qquad A = 30°$$

**13.** (c) Number of non-collinear points $= 18 - 5 = 13$

$\therefore$ Total number of straight lines $= ({}^{18}C_2 - {}^{5}C_2) + 1$

$$= 153 - 10 + 1$$

$$= 144$$

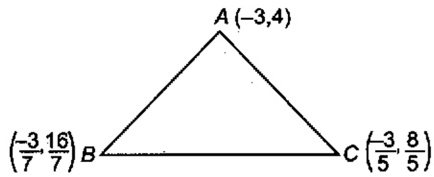
**14.** (a) Given lines

$$x + y = 1 \qquad \text{...(i)}$$

$$2x + 3y = 6 \qquad \text{...(ii)}$$

and $\qquad 4x - y + 4 = 0 \qquad$ ...(iii)

Solving these equations pairwise, we get vertices of triangle are $(-3, 4)$, $\left(\frac{-3}{7}, \frac{16}{7}\right)$ and $\left(\frac{-3}{5}, \frac{8}{5}\right)$.




Since, the triangle is scalene.

Hence, orthocentre lies in Ist quadrant.

**15.** (d) The line $4x + 11y - 15 = 0$ is included in the given system of straight lines.

As, $\qquad 2a + 3b = 4$

$$a + 2b = 11$$

gives $\qquad b = 18, a = -25$

Also, $\qquad -3a - 5b = -3(-25) - 5(18)$

$$= 75 - 90 = -15$$

Distance of $4x + 11y - 15 = 0$ from $(4, -3)$

$$= \left|\frac{16 - 33 - 15}{\sqrt{16 + 121}}\right| = \frac{32}{\sqrt{137}} = 2.73$$

## JNU MCA Solved Paper 2013

The line $7x + y - 8 = 0$ is included in the given system of straight lines.

$\therefore \quad 2a + 3b = 7$

$a + 2b = 1$

gives $\quad b = -5, a = 11$

Also, $\quad -3a - 5b = -3(11) - 5(-5)$

$= -33 + 25$

$= -8$

Distance of $7x + y - 8 = 0$ from $(4, -3)$

$$= \left| \frac{28 - 3 - 8}{\sqrt{49 + 1}} \right| = \frac{17}{\sqrt{50}} = 2.4$$

The line $4x + 3y - 7 = 0$ is included in the given system of straight lines.

$\therefore \quad 2a + 3b = 4$

and $\quad a + 2b = 3$

gives $\quad b = 2$ and $a = -1$

Also, $\quad -3a - 5b = -3(-1) - 5(2) = 3 - 10$

$= -7$

Distance of $4x + 3y - 7 = 0$ from $(4, -3)$

$$= \left| \frac{16 - 9 - 7}{\sqrt{16 + 9}} \right| = 0$$

The line $3x - 4y + 1 = 0$ is included in the given system of straight lines.

Distance of $3x - 4y + 1 = 0$ from $(4, -3)$

$$= \left| \frac{12 + 12 + 1}{\sqrt{9 + 16}} \right|$$

$$= \frac{25}{\sqrt{25}} = \frac{25}{5} = 5$$

**16.** (*d*) $\because A + B = \frac{\pi}{3}$

$$\therefore \quad \cos(A + B) = \cos \frac{\pi}{3}$$

$$\Rightarrow \quad \cos A \cos B - \sin A \sin B = \frac{1}{2}$$

$$\Rightarrow \quad \cos A \cos B = \frac{1}{2} + \sin A \sin B$$

Now, $\quad$ AM $\geq$ GM

$$\Rightarrow \quad \sec A + \sec B \geq 2\sqrt{\sec A \sec B}$$

$$\geq 2\sqrt{\frac{1}{\cos A \cos B}}$$

$$\Rightarrow \quad \sec A + \sec B \geq 2\sqrt{\frac{1}{\frac{1}{2} + \sin A \sin B}} \quad \text{...(i)}$$

For $\frac{1}{\frac{1}{2} + \sin A \sin B}$ to be minimum,

$\left(\frac{1}{2} + \sin A \sin B\right)$ should be maximum.

i.e. $\sin A \sin B$ should be maximum.

$$\Rightarrow \quad \sin A \sin B < 1$$

$$\Rightarrow \quad \frac{1}{2} + \sin A \sin B < \frac{3}{2}$$

From Eq. (i),

$$\sec A + \sec B \geq 2\sqrt{\frac{2}{3}} = \frac{2\sqrt{2}}{\sqrt{3}}$$

Hence, minimum value of $\sec A + \sec B$ is $\frac{2\sqrt{2}}{\sqrt{3}}$

**17.** (*b*) $(a^2 + b^2 + c^2)x^2 - 2x(ab + bc + cd) + b^2 + c^2 + d^2 \leq 0$

$\Rightarrow (a^2x^2 - 2xab + b^2) + (ax - b)^2 + (bx - c)^2 + (cx - d)^2 \leq 0$

$\Rightarrow \quad ax = b, bx = c, cx = d$

$\Rightarrow \quad x = \frac{b}{a}, x = \frac{c}{b}, x = \frac{d}{c}$

Hence, $a, b, c$ and $d$ are in GP.

**18.** (*a*) Given, $y^2 - x^2 + 2x - 1 = 0$ ...(i)

which is of the form $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$

where, $\quad a = -1, b = 1, h = 0, g = 1, f = 0$ and $c = -1$

$\therefore \quad abc + 2fgh - af^2 - bg^2 - ch^2$

$= 1 + 0 - 0 - 1 - 0 = 0$

Which represents a pair of straight line.

**19.** (*c*) Let $S = \{v_1, v_2, v_3, v_4\}$

where, $\quad v_1 = (2, -1, 0, 1), v_2 = (1, 2, -3, 2)$

$v_3 = (1, -3, 2, 0), v_4 = (0, 0, 1, -1)$

Let $\quad W = L(S) = L\{v_1, v_2, v_3, v_4\}$

Consider, $\alpha v_1 + \beta v_2 + \gamma v_3 + \delta v_4 = 0$

We get, $\quad \alpha = \beta = \delta = -\gamma$

$\therefore v_1, v_2, v_3$ and $v_4$ are not linear transformation.

Note that,

$v_1 = v_2 + v_3 + v_4$

$v_1 \in L\{v_2, v_3, v_4\}$

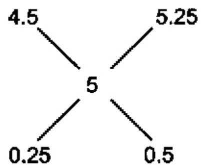
Now, it is easy to verify.

$\Rightarrow \quad \alpha v_2 + \beta v_3 + \gamma v_4 = 0$

$\Rightarrow \quad \alpha = \beta = \gamma = 0$

$\therefore \quad \dim(w) = 3$

**20.** (*a*)




$\therefore$ Required ratio $= 0.25 : 0.5 = 25 : 50 = 1 : 2$

**21.** (*d*) $(p - 3)x^2 - 2px + 5p = 0$

As, $\quad A > 0, D \geq 0$

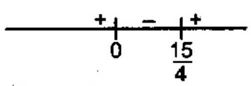
$\Rightarrow (-2p)^2 - 4(p - 3)(5p) \geq 0$

$\Rightarrow \quad 4p^2 - 20(p^2 - 3p) \geq 0$

$\Rightarrow \quad p^2 - 5(p^2 - 3p) \geq 0$

$\Rightarrow \quad p^2 - 5p^2 + 15p \geq 0 \Rightarrow -4p^2 + 15p \geq 0$

$\Rightarrow \quad p(15 - 4p) \geq 0 \Rightarrow p(4p - 15) \leq 0$




$\Rightarrow \quad p \in \left(0, \frac{15}{4}\right]$ ...(i)

$$A > 0$$

$\Rightarrow \quad p - 3 > 0$

$\Rightarrow \quad p > 3 \qquad \text{...(ii)}$

From Eqs. (i) and (ii), we get

$$p \in \left[3, \frac{15}{4}\right]$$

**22.** (*d*) $\lim_{x \to \infty} \frac{1 + x + x^2}{x(\ln x)^3} \qquad \left[\frac{\infty}{\infty} \text{ form}\right]$

$$= \lim_{x \to \infty} \frac{1 + 2x}{x \cdot 3(\ln x)^2 \cdot \frac{1}{x} + (\ln x)^3}$$

[using L'Hospital's rule]

$$= \lim_{x \to \infty} \frac{1 + 2x}{3(\ln x)^2 + (\ln x)^3} \qquad \left[\frac{\infty}{\infty} \text{ form}\right]$$

$$= \lim_{x \to \infty} \frac{1}{6(\ln x) \times \frac{1}{x} + 3(\ln x)^2 \times \frac{1}{x}}$$

$$= \lim_{x \to \infty} \frac{x}{6(\ln x) + 3(\ln x)^2} = \lim_{x \to \infty} \frac{1}{\frac{6}{x} + 6\frac{\ln x}{x}}$$

$$= \lim_{x \to \infty} \frac{x}{6 + 6\ln x} = \lim_{x \to \infty} \frac{1}{\frac{6}{x}} = \lim_{x \to \infty} \frac{x}{6} = \infty$$

**23.** (*d*) $2\sin^2\theta + 2\cos(\theta + \phi)[\cos(\alpha - \theta) - \cos(\alpha + \theta)] + \cos(2\alpha + 2\theta)$

$= 2\sin^2\theta + 2\cos(\theta + \phi)\cos(\alpha - \theta) - 2\cos(\theta + \phi)\cos(\alpha + \theta) + \cos 2(\alpha + \theta)$

$= 2\sin^2\theta + \cos(\phi + \alpha) + \cos(2\theta + \phi - \alpha) - \cos(2\theta + \alpha + \phi) - \cos(\phi - \alpha) + \cos 2(\alpha + \theta)$

$= 2\sin^2\theta + \cos(\phi + \alpha) - \cos(\phi - \alpha) + \cos(2\theta + \phi - \alpha) - \cos(2\theta + \alpha + \phi) + \cos 2(\alpha + \theta)$

$= 2\sin^2\theta - 2\sin\phi\sin\alpha - 2\sin(2\theta + \phi)\sin\alpha + \cos 2(\alpha + \theta)$

**24.** (*c*) (a) $\sin 15° = \frac{\sqrt{3} - 1}{2\sqrt{2}}$, which is irrational number.

(b) $\cos 15° = \frac{\sqrt{3} + 1}{2\sqrt{2}}$, which is irrational number.

(c) $\sin 15° \cos 15° = \frac{1}{2}(2\sin 15° \cos 15°)$

$= \frac{1}{2}\sin 30° = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, which is rational number.

**25.** (*d*) $c = \{z : |z| < 2\}$ is a circle with centre (0, 0) and radius 2.

Now, poles are given by putting denominator equal to zero.

$\Rightarrow \quad (z - 1)^3 = 0$

$\Rightarrow \quad z = 1, 1, 1$

Hence, triple pole at $z = 1$ which lies inside contour $C$, is

$$\int_C \left(\frac{z + 1}{z - 1}\right)^3 dz = \frac{2\pi i}{2!} \times f''(1)$$

$$\left[\because f''(z_0) = \frac{n!}{2\pi i} \int_C \frac{f(z)dz}{(z - z_0)^{n+1}}\right]$$

where, $f(z) = (z + 1)^3$

$$= \pi i \times 6(1 + 1) = 12\pi i$$

**26.** (*a*) First tap takes 18 min to do = 1 work

First tap takes 1 min to do $= \frac{1}{18}$ work

Second tap takes 24 min to do = 1 work

Second tap takes 1 min to do $= \frac{1}{24}$ work

In 12 min, the work done by second tap

$$= \frac{12}{24} = \frac{1}{2} \text{ work}$$

So, $\frac{1}{2}$ work is to be done by first tap in $\frac{1}{2} \times 18$ i.e. 9 min.

Hence, first tap is turned off after 9 min.

**27.** (*d*) Given, $\frac{\cos x}{a} = \frac{\sin x}{b}$

$\therefore \quad |a\cos 2x + b\sin 2x|$

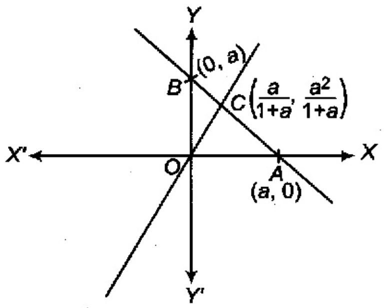
$= |a|\,|\cos 2x + \frac{b}{a}\sin 2x|$

$= |a|\,|\cos 2x + \frac{b}{a} \cdot 2\sin x \cos x|$

$= |a|\,|\cos 2x + 2\sin^2 x| \qquad \left[\because \frac{\cos x}{a} = \frac{\sin x}{b}\right]$

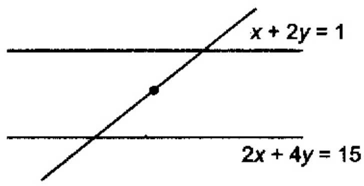
$= |a|\,|1 - 2\sin^2 x + 2\sin^2 x| = |a|$

**28.** (*d*)




$\therefore$ Area of $\Delta COB = \frac{1}{2} \times a \times \frac{a}{1 + a} = \frac{1}{2} \times \frac{a^2}{|1 + a|}$

**29.** (*b*) Let a variable point be $p \equiv \left(1 + \frac{\alpha}{\sqrt{2}}, 2 + \frac{\alpha}{\sqrt{2}}\right)$




Let $P$ be on $x + 2y = 1$. Then,

$$1 + \frac{\alpha}{\sqrt{2}} + 2\left(2 + \frac{\alpha}{\sqrt{2}}\right) = 1$$

or $\quad \alpha = -\frac{-4\sqrt{2}}{3}$

Let $P$ be on $2x + 4y = 15$. Then,

$$2\left(1 + \frac{\alpha}{\sqrt{2}}\right) + 4\left(2 + \frac{\alpha}{\sqrt{2}}\right) = 15$$

or $\quad \alpha = \frac{5\sqrt{2}}{6}$

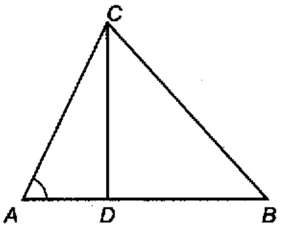
## JNU MCA Solved Paper 2013

Since, point lies between the lines and $x = \alpha$.

$\therefore \quad \alpha \in \left(\frac{-4\sqrt{2}}{3}, \frac{5\sqrt{2}}{6}\right)$

or $\quad \frac{-4\sqrt{2}}{3} < \alpha < \frac{5\sqrt{2}}{6}$

**30.** (*b*) We have, $\cot A = \frac{AD}{CD}$




and $\quad \cot B = \frac{DB}{CD}$

$\Rightarrow \quad \cot A + \cot B = \frac{AD + DB}{CD}$

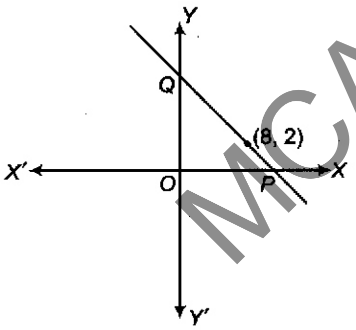
$\Rightarrow \quad \text{Constant} = \frac{AB}{CD}$

$\Rightarrow \quad CD = \text{Constant} \quad [\because AB = \text{constant}]$

So, the locus of $C$ is a straight line parallel to $AB$.

**31.** (*b*) Given slope is negative.

$\therefore \quad m = -k, k > 0$




$\therefore$ Equation of line passing through (8, 2) having slope $(-k)$ is

$$(y - 2) = -k(x - 8)$$

$\Rightarrow \quad kx + y - 2 - 8k = 0$

$\Rightarrow \quad \frac{kx}{2 + 8k} + \frac{y}{2 + 8k} = 1$

$\Rightarrow \quad \frac{x}{\frac{2 + 8k}{k}} + \frac{y}{2 + 8k} = 1$

$\Rightarrow \quad OP = \frac{2 + 8k}{k}, OQ = 2 + 8k$

Now, $\quad OP + OQ = \frac{2 + 8k}{k} + \frac{2 + 8k}{1}$

$= \frac{1}{k}(8k^2 + 10k + 2) = 8k + \frac{2}{k} + 10$

$= 2\left(k + \frac{1}{k}\right) + 4k + 10 \quad$ ...(i)

Now, $\quad$ AM $\geq$ GM

$\Rightarrow \quad k + \frac{1}{K} \geq 2\sqrt{k \cdot \frac{1}{k}}$

$\Rightarrow \quad k + \frac{1}{k} \geq 2$

Equality holds only when, $k = \frac{1}{k}$

$\Rightarrow \quad k = \pm 1$

$\Rightarrow \quad k = 1 \quad [\because k > 0]$

$\therefore OP + OQ = 2\left(k + \frac{1}{k}\right) + 4k + 10 \geq 2 \times 2 + 4 \times 1 + 10 \geq 18$

Hence, the minimum value of $(OP + OQ)$ is 18.

**32.** (*d*) The given lines meet the coordinate axes $P\left(-\frac{1}{2}, 0\right)$, $Q$ (0, 1), $R$ (3, 0) and $S$ $\left(0, \frac{3}{a}\right)$.

$\therefore$ Equation of the circle passing through $P$, $Q$ and $R$ is

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$\Rightarrow x^2 + y^2 + 2\left(\frac{-5}{4}\right)x + 2\left(\frac{1}{4}\right)y + (-3/2) = 0$

$\Rightarrow \quad x^2 + y^2 - \frac{5x}{2} + \frac{y}{2} - \frac{3}{2} = 0$

$\Rightarrow \quad 2x^2 + 2y^2 - 5x + y - 3 = 0$

Since, this circle passes through $S$ $\left(0, \frac{3}{a}\right)$

$\therefore \quad 2\left(\frac{3}{a}\right)^2 + \left(\frac{3}{a}\right) - 3 = 0$

$\Rightarrow \quad \frac{18}{a^2} + \frac{3}{a} - 3 = 0$

$\Rightarrow \quad \frac{4}{a^2} + \frac{1}{a} - 1 = 0$

$\Rightarrow \quad 6 + a - a^2 = 0$

$\Rightarrow \quad a^2 - a - 6 = 0$

$\Rightarrow \quad a^2 - 3a + 2a - 6 = 0$

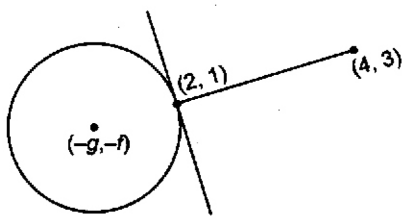
$\Rightarrow \quad a(a - 3) + 2(a - 3) = 0$

$\Rightarrow \quad (a - 3)(a + 2) = 0$

$\Rightarrow \quad a = -2 \text{ or } 3$

**33.** (*b*) Equation of normal at point (2,1) is

$$(1 + f)x - (2 + g)y + (g - 2f) = 0$$




Since, (4, 3) lies on this line.

So, $\quad 4(1 + f) - (2 + g)3 + (g - 2f) = 0$

$\Rightarrow \quad 2f - 2g - 2 = 0$

$\Rightarrow \quad f - g = 1 \quad$ ...(i)

Equation of the diameter is $2x + y = 2$.

As $(-g, -f)$ is the solution of this equation.

So, $-2g + f = 2$ ...(ii)

From Eqs. (i) and (ii),

$$g = -1 \text{ and } f = 0$$

Hence, centre $= (1, 0)$

$$\text{Radius } (r) = \sqrt{(1-0)^2 + (2-1)^2} = \sqrt{(1+1)} = \sqrt{2}$$

$\Rightarrow \quad r^2 = g^2 + f^2 - c$

$\Rightarrow \quad c = g^2 + f^2 - r^2 = 1 - 2 = -1$

Therefore, the equation of the circle is

$$x^2 + y^2 + 2(-1)x + 2(0)y + (-1) = 0$$

$\Rightarrow \quad x^2 + y^2 - 2x - 1 = 0$

**34.** (*) Power series

$$\sum_{n=0}^{\infty} \frac{(n!)^3}{3n!} z^n = \sum_{n=0}^{\infty} a_n z^n$$

$$R = \lim_{h\to\infty} \left| \frac{a_n}{a_{n+1}} \right|$$

$$= \lim_{h\to\infty} \left| \frac{(n!)^3}{3n!} \times \frac{3(n+1)!}{[(n+1)!]^3} \right|$$

$$= \lim_{h\to\infty} \left| \frac{(n!)^2}{[(n+1)!]^2} \right|$$

$$= \lim_{h\to\infty} \left| \frac{n!\,n!}{(n+1)^2 n!\,n!} \right|$$

$$= \lim_{h\to\infty} \frac{1}{n^2 + 2n + 1} = 0$$

(*) None of the option is correct.

**35.** (c) Sum of temperature on Monday, Tuesday and Wednesday $= 4 \times 3 = 12°C$

Sum of temperature on Tuesday, Wednesday and Thursday

$$= 5.5 \times 3 = 16.5°C$$

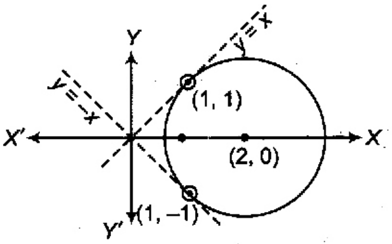
Temperature on Monday $= 2.6°C$

Temperature on (Tuesday + Wednesday) $= 12 - 2.6$

$$= 9.4°C$$

Temperature on Thursday $= 16.5 - 9.4 = 7.1°C$

**36.** (a, c) $x^2 + y^2 - 4x + 2 = 0$ is a circle whose centre is (2,0) and radius is $\sqrt{2}$ units.




When this circle touches $|y| = x$

Then, $\quad 2x^2 - 4x + 2 = 0$

$\Rightarrow \quad (x-1)^2 = 0$

$\Rightarrow \quad x = 1$

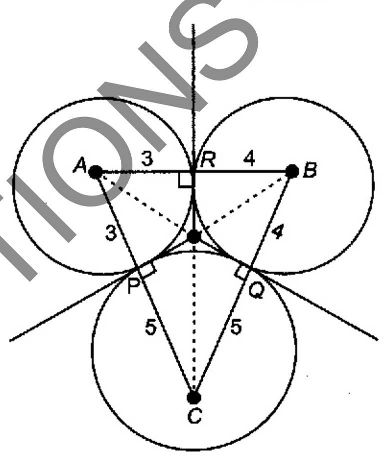
$\Rightarrow \quad y = \pm 1$

Both points (1, 1) and (1,−1) are at a distance of $\sqrt{2}$ units from the origin.

Similarly, in case of circle $x^2 + y^2 + 4x + 2 = 0$, we get the points (−1, 1) and (−1, −1) which are at a distance of $\sqrt{2}$ units from the origin.

**37.** (c) $\Delta ABC$ whose sides are

$$AB = 7 \text{ units}$$
$$BC = 9 \text{ units}$$
$$CA = 8 \text{ units}$$




Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{12(12-7)(12-9)(12-8)} \quad \left[\because s = \frac{7+9+8}{2} = \frac{24}{2} = 12\right]$$

$$= \sqrt{12 \times 5 \times 3 \times 4}$$

$$= 12\sqrt{5}$$

$\therefore$ Length of altitude $CR$

$$= \frac{2 \times \text{Area}}{AB} = \frac{2 \times 12\sqrt{5}}{7}$$

$$= \frac{24}{7}\sqrt{5} \text{ units}$$

Length of altitude $BP$

$$= \frac{2 \times \text{Area}}{AC} = \frac{2 \times 12\sqrt{5}}{8}$$

$$= 3\sqrt{5} \text{ units}$$

Length of altitude $AQ$

$$= \frac{2 \times \text{Area}}{BC} = \frac{2 \times 12\sqrt{5}}{9} = \frac{24}{9}\sqrt{5} \text{ units}$$

Here,

$$OP = OR = OQ = k\sqrt{5}$$

where, $k$ is a positive rational number.

If $\quad k = 1$

Then,

$$OP = OR = OQ = \sqrt{5} \text{ units}$$

**38.** (c) $C_1 \equiv x^2 + y^2 - 4x + 6y + 11 = 0$

Centre = (2, − 3)

Radius $(r_1) = \sqrt{4+9-11}$

$= \sqrt{2}$

$C_2 \equiv x^2 + y^2 - 2x + 8y + 13 = 0$

Centre = (1, − 4)

Radius $(r_2) = \sqrt{1+16-13}$

$= \sqrt{4} = 2$

So, distance $(d) = \sqrt{(-4+3)^2 + (1-2)^2}$

$= \sqrt{(-1)^2 + (-1)^2}$

$= \sqrt{2}$

$\cos\theta = \dfrac{r_1^2 + r_2^2 - d^2}{2r_1r_2}$

$= \dfrac{2+4-2}{2\times 2\sqrt{2}}$

$= \dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad \theta = 45°$

**39.** (d) $\dfrac{1}{z^2 - 5z + 4}$, $1 < |z| < 4$

$= \dfrac{1}{(z-4)(z-1)} = \dfrac{1}{3(z-4)} - \dfrac{1}{3(z-1)}$

$= \dfrac{1}{12\left(\frac{z}{4} - 1\right)} + \dfrac{1}{3(1-z)}$

$= -\dfrac{1}{12}\left(1 - \dfrac{z}{4}\right)^{-1} + \dfrac{1}{3}(1-z)^{-1}$

$= -\dfrac{1}{12}\left[1 + \dfrac{z}{4} - \dfrac{z^2}{16} + \dfrac{z^3}{64} + \dots\right] + \dfrac{1}{3}(1-z)^{-1}$

$\therefore$ Coefficient of $z^2 = -\dfrac{1}{12\times 16} = -\dfrac{1}{192}$

**40.** (b) Here, present value $(v)$ = ₹ 3310

Let each instalment = ₹$A$

$r = 10\%$

Number of instalments $(n) = 3$

$\because \quad v = \dfrac{A}{r}[1 - (1+r)^{-n}]$

$\Rightarrow \quad 3310 = \dfrac{A}{1}[1 - (1\cdot 1)^{-3}]$

$\Rightarrow \quad 331 = A[1 - (1\cdot 1)^{-3}]$

$\Rightarrow \quad 331 = A\left[\dfrac{(1\cdot 1)^3 - 1}{(1\cdot 1)^3}\right]$

$\Rightarrow \quad 331 = A\left[\dfrac{1.331 - 1}{1.331}\right]$

$\Rightarrow \quad 331 = \dfrac{A \times .331}{1.331}$

$\Rightarrow \quad A$ = ₹ 1331

Total amount paid in three instalments

= 1331 × 3 = ₹ 3993

$\therefore$ Total interest paid = 3993 − 3310 = ₹ 683

**41.** (c) $(101)^{100} - 1$

$= (1 + 100)^{100} - 1$

$= \left(1 + 100\times 100 + 100\times 99 \times \dfrac{(100)^2}{2!} + \dots\right) - 1$

$= \left(10000 + (100)^3 \times \dfrac{99}{2!} + \dots\right)$

$= 10000\left(1 + 100 \times \dfrac{99}{2!} + \dots\right)$

Hence, the greatest integer, which divides $(101)^{100} - 1$, is 10000.

**42.** (c) $(1 + 2\sqrt{x})^{40} = {}^{40}C_0 + {}^{40}C_1(2\sqrt{x}) + {}^{40}C_2(2\sqrt{x})^2 + {}^{40}C_3(2\sqrt{x})^3 + \dots + {}^{40}C_{40}(2\sqrt{x})^{40}$

$\therefore$ Sum of coefficients of all integral powers of $x$

$= {}^{40}C_0 + {}^{40}C_2\cdot 2 + {}^{40}C_4\cdot(2)^2 + \dots + {}^{40}C_{40}\times(2)^{40}$

$= \dfrac{1}{2}(3^{40} - 1)$

**43.** (a) $\sum_{r=0}^{n}\sum_{s=0}^{n}(C_r + C_s)$

$= \sum_{r=0}^{n}\left(\sum_{s=0}^{n}{}^nC_r + \sum_{s=0}^{n}{}^nC_s\right)$

$= \sum_{r=0}^{n}{}^nC_r\sum_{s=0}^{n}1 + 2^n$

$= \sum_{r=0}^{n}{}^nC_r\,(n+1) + 2^n$

$= (n+1)\sum_{r=1}^{n}{}^nC_r + 2^n\sum_{s=0}^{n}1$

$= (n+1)2^n + 2^n(n+1)$

$= 2(n+1)2^n$

$= (n+1)2^{n+1}$

**44.** (d) It is obvious that the options (a), (b) and (c) are properties of ${}^nC_r$.

Hence, option (d) is not the property of ${}^nC_r$

**45.** (b) Required number of ways

$= 12 \times 11 = 132$ ways

**46.** (b) Let number of 5 paisa coins be $3k$.

Number of 10 paisa coins be $2k$.

and number of 20 paisa coins be $k$.

$\therefore \quad 3k \times 5 + 2k \times 10 + 20k = 1100$

$\Rightarrow \quad 55k = 1100$

$\Rightarrow \quad k = \dfrac{1100}{55} = 20$

$\therefore$ Number of 5 paisa coins = 3 × 20 = 60

**47.** (c) Required number of ways $= 4^5$

**48.** (b) Circumradius, $R = \dfrac{abc}{4\Delta}$

Here, $a = 3, b = 4$ and $c = 5$

So $s = \dfrac{a+b+c}{2} = \dfrac{12}{2} = 6$ units

$\therefore \Delta = \sqrt{6(6-3)(6-4)(6-5)}$

$= \sqrt{6 \times 3 \times 2 \times 1} = 6$ sq units

Also, $R = \dfrac{3 \times 4 \times 5}{4 \times 6} = \dfrac{15}{6} = 2.5$ units

**49.** (c) The sides of a triangle are 17, 25 and 28.

Area of triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

where, $s = \dfrac{17+25+28}{2} = \dfrac{70}{2} = 35$

$\therefore$ Area $= \sqrt{35(35-17)(35-25)(35-28)}$

$= \sqrt{35 \times 18 \times 10 \times 7}$

$= \sqrt{7 \times 5 \times 9 \times 2 \times 2 \times 5 \times 7}$

$= 7 \times 5 \times 2 \times 3 = 210$

Length of altitudes

(i) $\dfrac{2 \times 210}{17} = \dfrac{420}{17}$

(ii) $\dfrac{2 \times 210}{25} = \dfrac{420}{25}$

(iii) $\dfrac{2 \times 210}{28} = \dfrac{420}{28}$

Hence, the greatest length of altitude is $\dfrac{420}{17}$.

**50.** (c) Given,

$y^2 = 4a(x+a)$ ...(i)

$y = mx + c$ ...(ii)

$\Rightarrow x = \left(\dfrac{y-c}{m}\right)$ ...(iii)

Eliminating $x$ from Eqs. (i) and (iii), we get

$y^2 = 4a\left(\dfrac{y-c}{m} + a\right)$

$\Rightarrow my^2 = 4a(y-c) + 4a^2m$

$\Rightarrow my^2 = 4ay - 4ac + 4a^2m$

$\Rightarrow my^2 - 4ay + 4ac - 4a^2m = 0$ ...(iv)

If line (ii) touches the parabola (i), then discriminant of Eq. (iv) must be zero.

$\therefore \Delta = 0$

$\Rightarrow (-4a)^2 - 4 \times m(4ac - 4a^2m) = 0$

$\Rightarrow 16a^2 = 4m(4ac - 4a^2m)$

$\Rightarrow 16a^2 = 16mac - 16a^2m^2$

$\Rightarrow 16a^2(1+m^2) = 16mac$

$\Rightarrow c = \dfrac{a^2(1+m^2)}{ma} = \dfrac{a(1+m^2)}{m}$

$\therefore c = am + a/m$

**51.** (b) $81x^2 + 144y^2 = 1944$

$\Rightarrow \dfrac{81x^2}{1944} + \dfrac{144y^2}{1944} = 1$

$\Rightarrow \dfrac{x^2}{24} + \dfrac{y^2}{\frac{27}{2}} = 1$

$\Rightarrow a^2 = 24, b^2 = \dfrac{27}{2} \Rightarrow a = 2\sqrt{6}$

Now, $SP + S'P = 2a = 2 \times 2\sqrt{6} = 4\sqrt{6}$

**52.** (a) Let the coordinates of the point be $(h, k)$.

Distance between $(-2, 0)$ and $(h, k)$ is

$d_1 = \sqrt{(h+2)^2 + k^2}$ ...(i)

and the distance of the point $(h, k)$ from the line $x = -9/2$ is

$d_2 = \dfrac{h + 9/2}{\sqrt{1^2 + 0^2}} = h + 9/2$ ...(ii)

According to the question,

$\sqrt{(h+2)^2 + k^2} = \dfrac{2}{3}(h + 9/2)$

$\Rightarrow (h+2)^2 + k^2 = \dfrac{4}{9}(h + 9/2)^2$

$\Rightarrow h^2 + 4 + 4h + k^2 = \dfrac{4}{9}\left(h^2 + \dfrac{81}{4} + 2 \times h \times \dfrac{9}{2}\right)$

$\Rightarrow h^2 + k^2 + 4h + 4 = \dfrac{4}{9}h^2 + 9 + 4h$

$\Rightarrow 9h^2 + 9k^2 + 36h + 36 = 4h^2 + 81 + 36h$

$\Rightarrow 9h^2 - 4h^2 + 9k^2 + 36 = 81$

$\Rightarrow 5h^2 + 9k^2 = 45$

$\Rightarrow \dfrac{5h^2}{45} + \dfrac{9k^2}{45} = 1$

$\Rightarrow \dfrac{h^2}{9} + \dfrac{k^2}{5} = 1 \Rightarrow \dfrac{x^2}{9} + \dfrac{y^2}{5} = 1$

which is an ellipse.

**53.** (d) $\lim\limits_{n \to \infty} (b^n + a^n)^{1/n}$

$= \lim\limits_{n \to \infty} b\left[1 + \left(\dfrac{a}{b}\right)^n\right]^{1/n} = b \lim\limits_{n \to \infty}\left[1 + \left(\dfrac{a}{b}\right)^n\right]^{1/n}$

$= b \cdot e^{\lim\limits_{n \to \infty}\left(\frac{a}{b}\right)^n \times \frac{1}{n}} = \infty$

**54.** (a) $\lim\limits_{x \to 0}\left[\dfrac{\sin x}{x}\right]$

$= \left[\lim\limits_{x \to 0} \dfrac{\sin x}{x}\right] = [1] = 1$

**55.** (b) Let $x^n = t$

$\Rightarrow nx^{n-1}dx = dt$

$\therefore \lim\limits_{n \to \infty} \int_0^1 \dfrac{dt}{1 + t^{1/x}} = \lim\limits_{n \to \infty} \int_0^1 \dfrac{dt}{1 + t^{1/n}}$

$= \int_0^1 \dfrac{dt}{1 + t^0} = \int_0^1 dt = [t]_0^1 = 1$

## JNU MCA Solved Paper 2013

**56.** (*d*) Let $CP$ be $x$.

Then, marked price $= \left(\frac{100+20}{100}\right) x = \frac{12}{10} x = 1.2\,x$

$\because$ Discount = 8%

$\therefore$ $SP = \left(\frac{100-8}{100}\right) \times \frac{12x}{10}$

$= \frac{92}{100} \times \frac{12}{10} \times x = 1.104\,x$

So, profit $= 1.104\,x - x = 0.104\,x$

$\therefore$ Profit % $= \frac{0.104\,x}{x} \times 100 = 10.4$

**57.** (*c*) Given, $A + B + C = \pi$

$\cos A + \cos B + \cos C$ will be maximum, when

$A = B = C$

$\Rightarrow$ $A = \pi/3 = B = C$

$\therefore \cos A + \cos B + \cos C = \cos\frac{\pi}{3} + \cos\frac{\pi}{3} + \cos\frac{\pi}{3}$

$= 3 \times \cos\frac{\pi}{3} = 3 \times \frac{1}{2} = 3/2$

**58.** (*a*) $f(x) = \begin{cases} 1, & x = 0, \frac{1}{n!}, \frac{2}{n!}, \dots, \frac{n}{n!}, n \in N \\ 0, & \text{otherwise} \end{cases}$

$f$ is discontinuous only at countable number of points.

$\Rightarrow f$ is Reimann integrable on [0, 1].

Similarly,

$$g(x) = \begin{cases} 1, & x = \frac{1}{n}, n \in N \\ 0, & \text{otherwise} \end{cases}$$

$g$ is discontinuous only at countable number of points.

$\Rightarrow g$ is also Riemann integrable on [0, 1].

**59.** (a) Between $x$ and $(x+1)$ O' clock, the two hands will coincide at

$$5 \times x \times \left(\frac{60}{55}\right) \text{ min past } x.$$

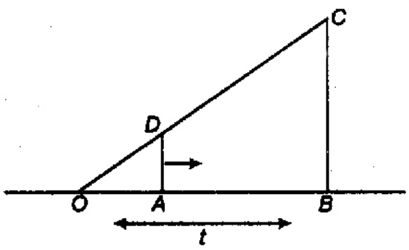
Hence, between 6 and 7 O' clock, the two hands will coincide at

$$5 \times 6 \times \left(\frac{60}{55}\right) \text{ min past 6,}$$

i.e. $\frac{360}{11} = 32$ min past 6.

Hence, both hands of clock coincide at 6 : 32.

**60.** (*b*) $d\frac{(OA)}{dt} = 0.1t$ m/s




Now, $\Delta OAD$ and $\Delta OBC$ are similar, hence

$$\frac{OA}{OB} = \frac{AD}{BC}$$

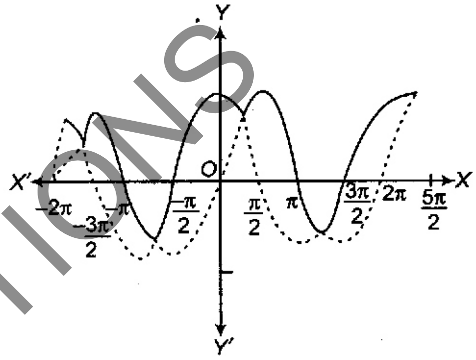
$\Rightarrow$ $BC = \frac{AD \times OB}{OA}$

$\Rightarrow$ $BC = \frac{mt}{OA}$

$\therefore$ $\frac{dBC}{dt} = \frac{-mt}{(OA)^2} \cdot \frac{dOA}{dt} = -\frac{mt}{t^2/4}(0.1t)$

$= -0.4\,m$ m/s

**61.** (*b*) $f(x) = \max(\sin x, \cos x)$




It is clear from the graph that,

Total number of critical points are 4.

**62.** (*d*) Average weight of combined group

$$= \frac{7 \times 66 + 5 \times 64}{12} = \frac{782}{12} = 65.17 \text{ kg}$$

**63.** (*c*) $P(x) = ax^3 + bx^2 + cx + d$

where $a, b, c, d \in R$

and $a \neq 0$

$\Rightarrow P'(x) = 3ax^2 + 2bx + c$

$\therefore$ $Q(x) = P(x) - \sqrt{2}P'(x)$

$= ax^3 + bx^2 + cx + d - \sqrt{2}(3ax^2 + 2bx + c)$

$= ax^3 + (b - 3\sqrt{2}a)x^2 + (c - 2b\sqrt{2})x + (d - \sqrt{2}c)$

Hence, $Q(x) = 0$ has exactly one real root and other complex roots are in conjugate pair.

**64.** (*a*) Volume of rectangular tank $= 4 \times 3 \times 1.5 = 18$ m$^3$

Volume of one stone $= 0.15 \times 0.1 \times 0.08 = 0.0012$ m$^3$

Volume of water raised by dropping stones in the tank

$$= 4 \times 3 \times 0.9 = 10.8 \text{ m}^3$$

$\therefore$ Number of stones $= \frac{10.8}{0.0012} = 9000$

**65.** (*c*) $f(x) = 2x - \tan^{-1} x - \ln(x + \sqrt{1+x^2}), \forall x \in R$

$\Rightarrow f'(x) = 2 - \frac{1}{1+x^2} - \frac{1}{x + \sqrt{1+x^2}} \times \left(1 + \frac{1 \times 2x}{2\sqrt{1+x^2}}\right)$

$= 2 - \frac{1}{1+x^2} - \frac{1}{\sqrt{1+x^2}} = \frac{2 - 2x^2 - 1}{1+x^2} - \frac{1}{\sqrt{1+x^2}}$

$$= \frac{(1-2x^2)\sqrt{1+x^2} - 1 - x^2}{(1+x^2)^{3/2}}$$

$$= \frac{(1-2x^2)\sqrt{1+x^2} - (1+x^2)}{(1+x^2)^{3/2}}$$

$$= \frac{\sqrt{1+x^2}\,(1-2x^2-\sqrt{1+x^2})}{(1+x^2)^{3/2}} = \frac{(1-2x^2)-\sqrt{1+x^2}}{(1+x^2)}$$

$\Rightarrow f'(x) > 0$ for all $n$.

$\therefore f(x)$ is increasing in $(-\infty, \infty)$.

**66.** (*d*) $\because \quad x^2 - x - 1 = 0$

$\Rightarrow \quad \alpha + \beta = 1$

and $\quad \alpha\beta = -1$

Now, $\frac{A_n + A_{n-1}}{2} = \frac{\alpha^n + \beta^n + \alpha^{n-1} + \beta^{n-1}}{2}$

$$= \frac{1}{2}(\alpha+\beta)(\alpha^{n-1}\beta + \alpha^{n-2}\beta^2 + \ldots + \alpha\beta^{n-1} + \alpha^{n-1} + \beta^{n-1})$$

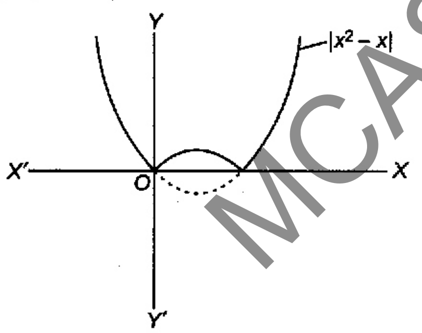
$$= \frac{1}{2}(\alpha^{n-1}\beta + \alpha^{n-2}\beta^2 + \ldots + \alpha\beta^{n-1} + \alpha^{n-1} + \beta^{n-1})$$

$$= \frac{1}{2}(-\alpha^{n-2} - \alpha^{n-4} + \ldots + \beta^{n-2} + \alpha^{n-1} + \beta^{n-1})$$

Now, we see that

$$\frac{A_n + A_{n-1}}{2} \neq 2A_{n+1}, \frac{1}{2}A_{n+1} \text{ or } 2A_{n-2}$$

**67.** (*d*) $f(x) = |x^2 - x|$




$$\left(y + \frac{1}{4}\right) = \left(x - \frac{1}{2}\right)^2$$

Equation of normal is

$$y + \frac{1}{4} = m\left(x - \frac{1}{2}\right) + \frac{2}{4} + \frac{1}{4m^2} \quad \ldots\text{(i)}$$

Since, point $(-2, 6)$ lies on Eq. (i).

$$\therefore \quad 6 + \frac{1}{4} = m\left(-2 - \frac{1}{2}\right) + \frac{1}{2} + \frac{1}{4m^2}$$

$$\Rightarrow \quad \frac{25}{4} = -\frac{5m}{2} + \frac{1}{4m^2} + \frac{1}{2}$$

$$\Rightarrow \quad \frac{25}{2} = -5m + \frac{1}{2m^2} + 1$$

$$\Rightarrow \quad \frac{23}{2} = -5m + \frac{1}{2m^2}$$

$$\Rightarrow \quad \frac{23}{2} = \frac{-10m^3 + 1}{2m^2}$$

$$\Rightarrow \quad 23m^2 = -10m^3 + 1$$

$$\Rightarrow \quad 10m^3 + 23m^2 - 1 = 0$$

Hence, no real value of $m$ exist.

**68.** (*b*) **Divisibility by 5**

If number having 0 or 5 at the end are divisible by 5, so 10.9876543210 is divisible by 5.

**Divisibility by 9**

If the sum of all the digits of a number is divisible by 9, then

$$9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$$

Clearly, 45 is divisible by 9.

**Divisibility by 11**

If the sum of digits at odd and even places are equal and differ by a number divisible by 11. So, it is not divisible by 11. Hence, option (d) is correct i.e. divisible by 5 and 9 but not by 11.

**69.** (*b*) Given, $23^{49} \times 51^{49}$

Unit's place of $23^{49}$ is $3^{49} = 3^{16} \times 3^1$

$= 3$

Unit's place of $51^{49}$ is $1^{49} = 1$

So, $\quad 3 \times 1 = 3$

**70.** (c) $P(x)Q(x)$

$= (ax^2 + bx + c)(-ax^2 + bx + c)$

$= -a^2x^4 + (ab - ab)x^3 + (ac + b^2 - ac)x^2 + (bc + bc)x + c^2$

$= -a^2x^4 + b^2x^2 + 2bcx + c^2$

Now,

$$P(x)Q(x) = 0$$

$$\Rightarrow \quad -a^2x^4 + b^2x^2 + 2bcx + c^2 = 0$$

$$\Rightarrow \quad a^2x^4 - b^2x^2 - 2bcx - c^2 = 0$$

Since, the coefficient of first term is positive and last term is negative.

So, the equation of degree 4 has atleast two real roots, one positive and one negative.

**71.** (*a*) Given $\quad a_0 > 0$

$a_1 > 0$

and $\quad a_2 = \sqrt{a_0} + \sqrt{a_1} > 0$

Then, $\quad a_3 = \sqrt{a_2} + \sqrt{a_1} = \sqrt{\sqrt{a_0} + \sqrt{a_1}} + \sqrt{a_1} > 0$

$a_4 = \sqrt{a_3} + \sqrt{a_2}$

$= \sqrt{\sqrt{\sqrt{a_0} + \sqrt{a_1}} + \sqrt{a_1}} + \sqrt{\sqrt{a_0} + \sqrt{a_1}} > 0$

Also, $\quad a_4 > a_3 > a_2 > a_1 > a_0$

It is a monotonically increasing sequence and not bounded above.

So, the squence does not converge.

**72.** (*d*) Total population of town = 24000

Let the population of town be $x$.

Then, the population of female become $(24000 - x)$.

After 6% increase in population of male and 9% increase in population of female, the total population become 25620.

As per the given condition,

$$\frac{106}{100} \times x + (24000 - x) \times \frac{109}{100} = 25620$$

$$\Rightarrow \quad 106x + (24000 - x) \times 109 = 2562000$$

$$\Rightarrow \quad 106x + 2616000 - 109x = 2562000$$

$$\Rightarrow \quad -3x = -54000$$

$$\Rightarrow \quad x = 18000$$

$\therefore$ Female population = 24000 − 18000 = 6000

and male population = 18000

**73.** (*d*) Given, $\sin^6\theta + \cos^6\theta + k\cos^2 2\theta = 1$

$$\Rightarrow (\sin^2\theta + \cos^2\theta)[\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta] + k\cos^2 2\theta = 1$$

$$\Rightarrow 1 \times [(\sin^2\theta + \cos^2\theta) - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta] + k\cos^2 2\theta = 1$$

$$\Rightarrow \quad 1 - 3\sin^2\theta\cos^2\theta + k\cos^2 2\theta = 1$$

$$\Rightarrow \quad \frac{3}{4}[4 \cdot \sin^2\theta\cos^2\theta] = k \cdot \cos^2 2\theta$$

$$\Rightarrow \quad \frac{3}{4}\sin^2 2\theta = k \cdot \cos^2 2\theta$$

$$\Rightarrow \quad \frac{3}{4}\tan^2 2\theta = k$$

**74.** (*b*) Given, sum in 2nd year = 6050

and sum in 3rd year = 6650

Then, interest = 6650 − 6050 = 605

$$\because \quad \text{Interest} = \frac{P \times R \times T}{100}$$

$$\therefore \quad 605 = \frac{6050 \times R \times 1}{100}$$

$$\Rightarrow \quad 60500 = 6050 \times R$$

$$\Rightarrow \quad R = 10\%$$

For sum, principal = $P$

Amount = ₹ 6050

Time period = 2 yr

Rate of interest = 10%

$$\therefore \quad 6050 = P\left(1 + \frac{10}{100}\right)^2 \Rightarrow 6050 = P \times \left(\frac{11}{10}\right)^2$$

$$\Rightarrow \quad P = \frac{6050 \times 100}{121} = 5000 \Rightarrow P = ₹\,5000$$

**75.** (*b*) 124 days before

$$\frac{124}{7} = 17 \text{ weeks and 5 days}$$

Today is Tuesday. So, 124 days before Tue ↓ Today is Mon, Sun, Sat, Fri, Turs ↓ Req. day

**76.** (*c*) We know that,

$$F_n(x) = a_0 + \sum_{k=1}^{n}(a_k \cos kx + b_k \sin kx)$$

As $\quad a_0 = \int_{-4}^{4}|x|\,dx = 0$, in $[-4, 4]$

$$\therefore \quad F_n(x) = \sum_{k=1}^{n}(a_k \cos kx + b_k \sin kx)$$

contains no constant term, both the sine and cosine terms.

**77.** (*b*) $f(x) - g(x) = x^2 - 4x - 3x + 6$

$$= x^2 - 7x + 6$$

$\therefore \sup\{|f(x) - g(x)|\} = \sup\{|x^2 - 7x + 6|\}$

Let $\quad y_1 = x^2 - 7x + 6$

and $\quad y_2 = -(x^2 - 7x + 6)$

$$\Rightarrow \quad \frac{dy_1}{dx} = 2x - 7$$

$$\Rightarrow \quad x = \frac{7}{2}$$

$$\Rightarrow \quad \frac{d^2y_1}{dx^2} = 2, +\text{ve}$$

$\therefore \sup\{|f(x) - g(x)|\}$ at $x = \frac{7}{2}$

i.e. $\left|\left(\frac{7}{2}\right)^2 - 7\left(\frac{7}{2}\right) + 6\right|$

$$= \left|\frac{49 - 98 + 24}{4}\right| = \left|\frac{-49 + 24}{4}\right| = \left|\frac{-25}{4}\right| = 6.25$$

**78.** (*b*) Volume of 3 cm cube $= (3)^3$

$= 27\text{ cm}^3$ [$\because$ volume of cube $= (a)^3$]

Volume of 4 cm cube $= (4)^3$

$= 64\text{ cm}^3$

Volume of 5 cm cube $= (5)^3$

$= 125\text{ cm}^3$

Total volume = 27 + 64 + 125

$= 216\text{ cm}^3$

Since, 216 cm$^3$ volume melted to form a new cube.

$$\therefore \quad 216 = (a)^3$$

$$\Rightarrow \quad (6)^3 = (a)^3$$

$$\Rightarrow \quad a = 6\text{ cm}$$

**79.** (*c*) Given, coding

| B | A | T | C | H |
|---|---|---|---|---|
| ↓ −1 | ↓ +1 | ↓ −1 | ↓ +1 | ↓ −1 |
| A | B | S | D | G |

Similarly,

| T | E | R | M |
|---|---|---|---|
| ↓ −1 | ↓ +1 | ↓ −1 | ↓ +1 |
| S | F | Q | N |

**80.** (*d*) Given, $g(a+b-x)=g(x)$

We know that,

$$I=\int_a^b x\,[g(x)]\,dx \quad \text{...(i)}$$

$$\Rightarrow \quad I=\int_a^b (a+b-x)\,g\,(a+b-x)\,dx$$

$$\Rightarrow \quad I=(a+b-x)\int_a^b g(a+b-x)\,dx \quad \text{...(ii)}$$

Adding Eqs. (i) and (ii), we get

$$2I=(a+b)\int_a^b g(a+b-x)\,dx$$

$$\Rightarrow \quad I=\frac{(a+b)}{2}\int_a^b g(x)\,dx$$

**81.** (*d*) None of the five men selects his own hat is

$$P(A)=\frac{5!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right]}{5!}$$

$$=\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}$$

**82.** (*d*) Given differential equation is

$$(x-y)(dx+dy)=dx-dy$$

$$\Rightarrow xdx+xdy-ydx-ydy=dx-dy$$

$$\Rightarrow \quad (x-y-1)dx=(y-x-1)dy$$

$$\Rightarrow \quad \frac{dy}{dx}=\left(\frac{x-y-1}{y-x-1}\right)$$

Put $\quad x-y=z \Rightarrow 1-\frac{dy}{dx}=\frac{dz}{dx} \Rightarrow \frac{dy}{dx}=1-\frac{dz}{dx}$

$$\therefore \quad 1-\frac{dz}{dx}=\frac{z-1}{-(z+1)}$$

$$\Rightarrow \quad \left(1+\frac{z-1}{z+1}\right)=\frac{dz}{dx}$$

$$\Rightarrow \quad \left(\frac{2z}{z+1}\right)=\frac{dz}{dx}$$

$$\Rightarrow \quad \int dx=\frac{1}{2}\left[\int \frac{z+1}{z}\,dz\right]=\frac{1}{2}\left[\int\left(1+\frac{1}{z}\right)dz\right]$$

$$\Rightarrow x=\frac{1}{2}[z+\log z]+C=\frac{1}{2}[(x-y)+\log|(x-y)|+C]$$

$y=-1$, when $x=0$

$$\Rightarrow \quad 0=\frac{1}{2}[1+0]+C \Rightarrow C=-1/2$$

$\therefore$ Solution is $x=\frac{1}{2}[x-y+\log|(x-y)|]+\frac{1}{2}$

$$\Rightarrow \quad 2x-x+y=\log|x-y|+1$$

$$\Rightarrow \quad x+y=\log|x-y|+1$$

$$\Rightarrow \quad x+y-1=\log|x-y|$$

**83.** (c) The planes $2x-3y+4z-5=0$ and $5x-2.5y-10z-6=0$ cut $Y$-axis at $y=-5/3$ and $y=\frac{-12}{5}$, respectively.

**84.** (c) $f(x)=\begin{cases}\dfrac{2-(1+x)^n}{n}, & -1<x<0\\ \dfrac{(1-x)^n}{n}, & 0<x<1\end{cases}$

Now, when $-1<x<0$, we have

$$0<x+1<1$$

$$\Rightarrow \quad (1+x)^n \to 0 \text{ as } n \to \le \infty$$

$$\therefore \quad \lim_{n\to\infty} f_n(x)=\lim_{n\to\infty}\frac{2-(1+x)^n}{n}=\lim_{n\to\infty}\frac{(1+x)^n}{n}=0$$

Thus, $f_n(x)\to 0$ as $n\to\infty$, $\forall\, x\in(-1,0)$

Then, $\quad 0>-x>-1$

$$\Rightarrow \quad 1>1-x<1 \Rightarrow 0<-x<1$$

$$\Rightarrow \quad (1-x)^n \to 0 \text{ and } n\to\infty$$

Then, $f_n(x)\to 0$ and $n\to\infty$, $\forall\, x\in(0,1)$

$\therefore \{f_n(x)\}$ converges on $(-1,1)$.

**85.** (*d*) Let the number be $x$.

Then, $x\times 9+9=17k$

If $x=12$, then $12\times 9+9=108+9=117$, not divisible by 17

If $x=15$, then $15\times 9+9=135+9=144$, not divisible

If $x=17$, then $17\times 9+9=153+9=162$, not divisible

If $x=16$, then $16\times 9+9=144+9=153$, divisible

**86.** (*a*) Following the definition of uniform probability distribution, for $\beta\le x\le\alpha$

$$f(x)=\begin{cases}0, & x\le\alpha\\ \dfrac{x-\alpha}{\beta-\alpha}, & \alpha<x<\beta\\ 1, & x\ge\beta\end{cases}$$

**87.** (*a*) Given equation is

$$x^2+y^2+z^2=1$$

$xyz$ will be maximum, when $x=y=z$.

i.e. $\quad 3x^2=1$

$$\Rightarrow \quad x=\frac{1}{\sqrt{3}} \Rightarrow y=z=\frac{1}{\sqrt{3}}$$

Hence, maximum value of $xyz$

$$=\left(\frac{1}{\sqrt{3}}\right)^3=\frac{1}{3\sqrt{3}}$$

**88.** (c) $I=\int \frac{1}{\sqrt{\sin^3 x\,\sin(x+\alpha)}}\,dx$

$$=\int \frac{1}{\sqrt{\sin^4 x\,(\cos x+\cot x\sin\alpha)}}\,dx$$

$$=\int \frac{\text{cosec}^2\,x}{\sqrt{\cos\alpha+\cot x\sin\alpha}}\,dx$$

$$=-\frac{1}{\sin\alpha}\int \frac{1}{\sqrt{\cos\alpha+\cot x\sin\alpha}}\,d(\cos\alpha+\cot x\sin\alpha)$$

$$=-\frac{2}{\sin\alpha}\cdot\sqrt{\cos\alpha+\cot x\sin\alpha}+C$$

$$=-\frac{2}{\sin\alpha}\cdot\sqrt{\frac{\sin(x+\alpha)}{\sin x}}+C$$

**89.** (*d*) The given series is not of $\infty$, $\pi$ or $e$.

**90.** (*a*) Let the third term be $x$.

So, first term $= \frac{7x}{4}$

Second term $= \frac{7x}{5}$

As per the given condition,

$$x + \frac{7x}{4} + \frac{7x}{5} = 830$$

$$\Rightarrow \frac{20x + 35x + 28x}{20} = 830$$

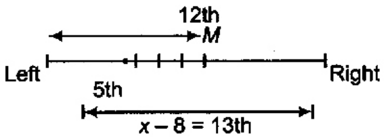
$$\Rightarrow \frac{83x}{20} = 830$$

$$\Rightarrow x = 200$$

$\therefore$ Third term = 200

So, first term $= \frac{7 \times 200}{4} = 7 \times 50 = 350$

**91.** (*d*)




After 12th position, there are 9 students.

So, position = 9 + 5 = 14th

**92.** (*a*) $343 = 7 \times 7 \times 7$

Two faces are painted.

$7 \times 7 = 49$

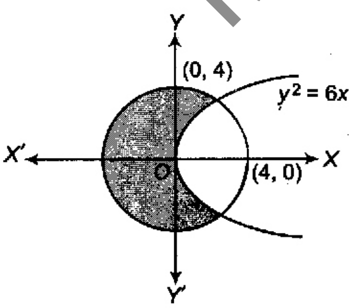
$7 \times 7 = 49$

$\therefore$ $49 + 49 = 98$ with black faces.

Not painted $= 343 - 98 = 245$

**93.** (*c*) Circle $\equiv x^2 + y^2 = 16$

Parabola $\equiv y^2 = 6x$




Now, $x^2 + 6x - 16 = 0$

$\Rightarrow x^2 + 8x - 2x - 16 = 0$

$\Rightarrow x = -8, 2$

Now, at $x = 2$,

$y = \pm\sqrt{12} = \pm 2\sqrt{3}$

Hence, both curves intersecting at $(2, 2\sqrt{3})$ and $(2, -2\sqrt{3})$.

$\therefore$ Area of shaded portion = Area of circle − Area of the circle interior to parabola

$$= \pi(4)^2 - 2\left[\int_0^2 \sqrt{6x}\,dx + \int_2^4 \sqrt{16 - x^2}\,dx\right]$$

$$= 16\pi - 2\left\{\sqrt{6}\left[\frac{x^{3/2}}{3/2}\right]_0^2 + \frac{1}{2}\left[x\sqrt{16 - x^2} + 16\sin^{-1}\left(\frac{x}{4}\right)\right]_2^4\right\}$$

$$= 16\pi - 2\left[\sqrt{6} \times \frac{2}{3}(2^{3/2})\right] + \frac{1}{2}\left[4 \times 0 + 16\sin^{-1}(1) - 2\sqrt{12} - 16\sin^{-1}\left(\frac{1}{2}\right)\right]$$

$$= 16\pi - 2\left[\frac{2}{\sqrt{3}} \times 4 + \frac{1}{2}\left(\frac{16 \times \pi}{2} - 2\sqrt{12} - \frac{16 \times \pi}{6}\right)\right]$$

$$= 16\pi - 2\left[8\frac{\sqrt{3}}{3} + \frac{1}{2}\left(8\pi - 2\sqrt{12} - 8\frac{\pi}{3}\right)\right]$$

$$= 16\pi - 2\left[\frac{8\sqrt{3}}{2} + \frac{1}{2}\left(\frac{16\pi}{3} - 2\sqrt{12}\right)\right]$$

$$= 16\pi - 2\left[\frac{8\sqrt{3}}{3} + \frac{8\pi}{3} - \sqrt{12}\right]$$

$$= 16\pi - 2\left[\frac{8}{3}\sqrt{3} - 2\sqrt{3} + \frac{8\pi}{3}\right] = 16\pi - 2\left[\frac{2}{3}\sqrt{3} + 8\frac{\pi}{3}\right]$$

$$= 16\pi - \frac{4}{3}\sqrt{3} - \frac{16\pi}{3} = \frac{32\pi}{3} - \frac{4\sqrt{3}}{3} = \frac{4(8\pi - \sqrt{3})}{3}$$

**94.** (*a*) We know that tangent to the curve $y^2 = 4x$ is $y = mx + \frac{A}{m}$

where, $A = 1$ for $y^2 = 4x$

$\Rightarrow$ $C = \frac{A}{m} = 1$ and $m = a$ [$\because$ tangent is $y = ax - 1$]

$\Rightarrow$ $\frac{1}{m} = -1 \Rightarrow m = -1 \Rightarrow a = -1$

**95.** (*d*) Given that, $X, Y$ and $Z$ are three independent random variables.

Here, $E(X) = E(Y) = E(Z) = \mu = 1, \sigma_x^2 = \sigma_y^2 = \sigma_z^2 = \sigma^2 = 0$

Now, $E[(X + Y + Z)^2] = E[X^2 + Y^2 + z^2 + 2(XY + YZ + ZX)]$

$= E(X^2) + E(Y^2) + E(Z^2) + 2E(XY + YZ + ZX)$

$= E(X^2) + E(Y^2) + E(Z^2) + E(XY) + 2E(YZ) + 2E(ZX)$

$= E(X^2) + E(Y^2) + E(Z^2) + 2E(X)E(Y) + 2E(Y)E(Z) + 2E(Z)E(X)$

$= (\sigma_x^2 + \mu^2) + (\sigma_y^2 + \mu^2) + (\sigma_z^2 + \mu^2) + 2\mu^2 + 2\mu^2 + 2\mu^2$

$= 3\sigma^2 + 3\mu^2 + 9\mu^2 = 3 \times 0 + 9\mu^2 = 3 \times 0 + 9 \times 1 = 9$

**96.** (*d*) Using binomial distribution,

$$P\text{ (50 heads)} = {}^{100}C_{50}\left(\frac{1}{2}\right)^{50}\left(\frac{1}{2}\right)^{50} = \frac{100!}{50!\,50!} \times \frac{1}{2^{100}} \quad \text{...(i)}$$

$\because P$ (50 heads) $\rightarrow P$ (51 heads)

$$\Rightarrow P\text{ (51 heads)} = {}^{100}C_{51}\left(\frac{1}{2}\right)^{51} \times \left(\frac{1}{2}\right)^{49} \quad \text{...(ii)}$$

$$\frac{P(51)}{P(50)} = \frac{{}^{100}C_{51} \cdot (1/2)^{51} \times (1/2)^{49}}{{}^{100}C_{50} \cdot (1/2)^{50} \times (1/2)^{50}}$$

$$= \frac{100!}{49! \times 51!} \Big/ \frac{100!}{50! \times 50!} \times \frac{1/2}{1/2}$$

$$= \frac{50! \times 50!}{49! \times 51!} = \frac{50}{51} \approx 0.9$$

**97.** (*a*) The total number of second digits number after decimal point between the interval (0, 1) = 99 = $n(S)$

[$\because$ 0.01, 0.02, ..., 0.10, ..., 0.99]

The total number of numbers which have 5 at second decimal digit = 10 = $n(E)$

[$\because$ 0.05, 0.15, 0.25, 0.35, 0.45, 0.55, 0.65, 0.75, 0.85, 0.95]

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{10}{99} = 0.1$

**98.** (*a*) Given, $f(x) = \frac{4^x}{4^x + 2}$

Then, $f(1-x) = \frac{4^{1-x}}{4^{1-x}+2} = \frac{4 \cdot 4^{-x}}{4 \cdot 4^{-x} + 2}$

$= \frac{4 \cdot 4^{-x} \cdot 4^x}{4 + 2 \cdot 4^x} = \frac{4}{2 \cdot 4^x + 2} = \frac{2}{4^x + 2}$

$\therefore f(x) + f(1-x) = \frac{4^x}{4^x+2} + \frac{2}{4^x+2} = \frac{4^x+2}{4^x+2} = 1$

Now, $f\left(\frac{1}{2001}\right) + f\left(\frac{2}{2001}\right) + \ldots + f\left(\frac{2000}{2001}\right)$

$= \left\{f\left(\frac{1}{2001}\right) + f\left(\frac{2000}{2001}\right)\right\} + \left\{f\left(\frac{2}{2001}\right) + f\left(\frac{1999}{2001}\right)\right\} + \ldots + \left\{f\left(\frac{1000}{2001}\right) + f\left(\frac{1001}{2001}\right)\right\}$

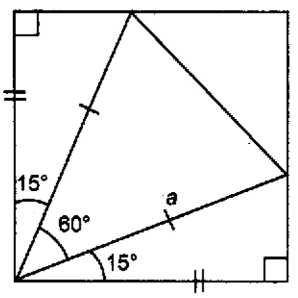
= (1) + (1) + ... + 1000 times = 1000

**99.** (*d*) $D_r = \begin{vmatrix} r & x & \frac{n(n+1)}{2} \\ 2r-1 & y & n^2 \\ 3r-2 & z & \frac{n(3n-1)}{2} \end{vmatrix}$

$\therefore \sum_{r=1}^{n} D_r = \begin{vmatrix} \Sigma r & x & \frac{n(n+1)}{2} \\ 2\Sigma r - 1 & y & n^2 \\ 3\Sigma r - 2 & z & \frac{n(3n-1)}{2} \end{vmatrix}$

$\Rightarrow \begin{vmatrix} \frac{n(n+1)}{2} & x & \frac{n(n+1)}{2} \\ n^2 & y & n^2 \\ \frac{3(n+1)n}{2} - 2 & z & \frac{n(3n-1)}{2} \end{vmatrix} = 0$

**100.** (*a*)




$\because \frac{1}{a} = \cos 15°$

$\Rightarrow a = \frac{1}{\cos 15°} = \frac{1}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$

$= \frac{2\sqrt{2}}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{2\sqrt{2}(\sqrt{3}-1)}{2} = \sqrt{2}(\sqrt{3}-1)$

$\therefore$ Area of triangle $= \frac{\sqrt{3}}{4} a^2$

$= \frac{\sqrt{3}}{4} \cdot 2(\sqrt{3}-1)^2$

$= \frac{\sqrt{3}}{2}(3 + 1 - 2\sqrt{3})$

$= \frac{\sqrt{3}}{2}(4 - 2\sqrt{3})$

$= \sqrt{3}(2 - \sqrt{3})$

$= (2\sqrt{3} - 3)$ sq unit

**101.** (*d*) Random variable $X$ can take the values of 0, 1, 2, 3, 4, 5, 6.

Then,

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $P(X)$ | 0 | $k$ | $2k$ | $3k$ | $2k$ | $k$ | 0 |

We know that, $\sum_{z=0}^{n} P(X) = 1$

Then, $k + 2k + 3k + 2k + k = 1$

$\Rightarrow 9k = 1$

$\Rightarrow k = 1/9$

**102.** (*d*) Let $I = \int \frac{dx}{x\sqrt{ax - x^2}}$

Putting $x = \frac{1}{t} \Rightarrow dx = -\frac{1}{t^2} dt$

$\therefore I = -\int \frac{dt}{t^2 \cdot \frac{1}{t}\sqrt{\frac{a}{t} - \frac{1}{t^2}}} = -\int \frac{dt}{\sqrt{at-1}}$

$= -\int (at-1)^{-1/2} dt = -2(at-1)^{1/2} + C$

$= -2\sqrt{\frac{a-x}{x}} + C$

**103.** (*d*) Let $I = \int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$

Put $(\cos x - \sin x)^2 = 1 - 2\sin x \cos x$

$\Rightarrow \sqrt{\sin 2x} = \sqrt{1 - (\cos x - \sin x)^2}$

$\therefore I = \int_{\pi/6}^{\pi/3} \frac{(\sin x + \cos x)}{\sqrt{1-(\cos x - \sin x)^2}} dx$

$= -[\{\sin^{-1}\cos x - \sin x\}]_{\pi/6}^{\pi/3} = \sin^{-1}(1 - \sqrt{3})$

**104.** (*a*) Quantity of milk in Ist mixture $= \frac{1}{6}$

Quantity of water in Ist mixture $= \frac{5}{6}$

Quantity of milk in 2nd mixture $= \frac{3}{8}$

Quantity of water in 2nd mixture $= \frac{5}{8}$

As per the given condition,

$$\frac{1}{6} + \frac{3}{8} = \frac{4+9}{24} = \frac{13}{24} \text{ milk}$$

and $$\frac{5}{6} + \frac{5}{8} = \frac{20+15}{24} = \frac{35}{24}$$

$\therefore$ Ratio of milk to water = 13 : 35

**105.** *(a)* Q runs 25 m is 5 s

Q runs 1000 m in $\frac{5}{25} \times 1000 = 200$ s

$\therefore$ P beats Q by 5 s.

P runs 1000 m in $200 - 5 = 195$ s

Time taken = 195 s

$= 3 \times 60 + 15 = 3$ min + 15 s

**106.** *(a)* Let the equation of the normal be $\frac{x}{a} + \frac{y}{a} = 1$

$\Rightarrow \quad x + y = a$

$\Rightarrow \quad y = -x + a$

Slope of the tangent is $\frac{x^2}{6y}$ at $(x, y)$.

$\Rightarrow \quad \frac{x^2}{6y}(-1) = -1$

$\Rightarrow \quad 36y^2 = x^4$

$\Rightarrow \quad 4x^3 - x^4 = 0$, gives $x = 4$

When $x = 4$, then $y = \pm 8/3$

Hence, the points of contact are $(4, \pm 8/3)$.

**107.** (c) Given, probability function

$$P(x = k) = \frac{\lambda^k}{(e^\lambda - 1)k!}, k = 1, 2, \ldots$$

$$\therefore \quad E(x) = \sum_{x=1}^{\infty} xP(x)$$

or $$E(x) = E(k) = \sum_{k=1}^{\infty} kP(k)$$

$$= \sum_{k=1}^{\infty} k \cdot \frac{\lambda^k}{(e^\lambda - 1)k!}$$

$$= \frac{1}{(e^\lambda - 1)} \sum_{k=1}^{\infty} \frac{k \cdot \lambda^k}{k!}$$

$$= \frac{1}{(e^\lambda - 1)} \sum_{k=1}^{\infty} \frac{\lambda^k}{(k-1)!}$$

$$= \frac{1}{(e^\lambda - 1)} \left\{ \frac{\lambda}{0!} + \frac{\lambda^2}{1!} + \frac{\lambda^3}{2!} + \ldots \right\}$$

$$= \frac{\lambda}{(e^\lambda - 1)} \left\{ \frac{1}{1} + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \ldots \right\}$$

$$= \frac{\lambda}{(e^\lambda - 1)} \left( 1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \ldots \right)$$

$$= \frac{\lambda \cdot e^\lambda}{(e^\lambda - 1)} = \frac{\lambda \cdot e^\lambda}{e^\lambda(1 - e^{-\lambda})} = \frac{\lambda}{(1 - e^{-\lambda})}$$

**108.** *(d)* Let the number be $x$.

As per the given condition,

$238k + 79 = (17 \times 14)k + (17 \times 4) + 11$

Hence, the number is divided by 17 gives remainder 11.

**109.** *(d)* Percentage loss by weight $= \frac{1000 - 990}{1000} \times 100$

$$= \frac{10}{1000} \times 100 = 1\%$$

$\therefore$ Net profit = 20% − 1%

= 19%

**110.** *(d)* Distance between two parallel planes is

$$d = \left| \frac{d_1 - d_2}{\sqrt{a^2 + b^2 + c^2}} \right| = \left| \frac{-4 + 6}{\sqrt{2^2 + 3^2 + 4^2}} \right|$$

$$= \frac{2}{\sqrt{4 + 9 + 16}} = \frac{2}{\sqrt{29}}$$