22

QUESTION PAPER SERIES CODE
A

Registration No. : | | | | | |

Centre of Exam. : ______________________

Name of Candidate : ______________________

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Signature of Invigilator

ENTRANCE EXAMINATION, 2014

MASTER OF COMPUTER APPLICATIONS

[Field of Study Code : MCAM (224)]

Time Allowed : 3 hours

Maximum Marks : 480
Weightage : 100

INSTRUCTIONS FOR CANDIDATES

Candidates must read carefully the following instructions before attempting the Question Paper :

(i) Write your Name and Registration Number in the space provided for the purpose on the top of this Question Paper and in the Answer Sheet.

(ii) **Please darken the appropriate Circle of Question Paper Series Code on the Answer Sheet.**

(iii) All questions are compulsory.

(iv) Answer all the 120 questions in the Answer Sheet provided for the purpose by darkening the correct choice, i.e., (a) or (b) or (c) or (d) with BALLPOINT PEN only against the corresponding circle. Any overwriting or alteration will be treated as wrong answer.

(v) Each correct answer carries **4** marks. **There will be negative marking and 1 mark will be deducted for each wrong answer.**

(vi) Answer written by the candidates inside the Question Paper will not be evaluated.

(vii) Calculators and Log Tables may be used.

(viii) Pages at the end have been provided for Rough Work.

(ix) Return the Question Paper and Answer Sheet to the Invigilator at the end of the Entrance Examination. **DO NOT FOLD THE ANSWER SHEET.**

INSTRUCTIONS FOR MARKING ANSWERS

1. Use only Blue/Black Ballpoint Pen (do not use pencil) to darken the appropriate Circle.
2. Please darken the whole Circle.
3. Darken ONLY ONE CIRCLE for each question as shown in the example below :

Wrong	Wrong	Wrong	Wrong	Correct
● ⓑ ⓒ ●	ⓐ̸ ⓑ ⓒ ⓓ	ⓐ̸ ⓑ ⓒ ⓓ̸	⦿ ⓑ ⓒ ●	ⓐ ⓑ ⓒ ●

4. Once marked, no change in the answer is allowed.
5. Please do not make any stray marks on the Answer Sheet.
6. Please do not do any rough work on the Answer Sheet.
7. Mark your answer only in the appropriate space against the number corresponding to the question.
8. **Ensure that you have darkened the appropriate Circle of Question Paper Series Code on the Answer Sheet.**

1. In the space of $n \times n$ matrix, let A and B be two matrices and let I denote the identity matrix. In relation to the function δ denoting the determinant, which one of the following statements is the incorrect one?

(a) $\delta(AB) = \delta(A)\,\delta(B)$

(b) $\delta(I) = 0$

(c) If row i of A is equal to a multiple of row j and $i \neq j$, then $\delta(A) = 0$

(d) If E is the elementary matrix of the second kind (row interchange), then $\delta(E) = 1$

2. Consider a cube $ABCDA_1B_1C_1D_1$ with the lower base $ABCD$, the upper base $A_1B_1C_1D_1$, and the lateral edges AA_1, BB_1, CC_1, DD_1. Let M and M_1 be the centres of the lower and upper bases respectively. If O is a point on the line MM_1 such that

$$OA + OB + OC + OD = OM$$

what is the value of λ if $OM = \lambda OM_1$?

(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) $\frac{1}{6}$

(d) $\frac{1}{8}$

3. If the nth difference of a tabulated function is constant when the values of the independent variables are at equal intervals apart, then the function is a

(a) constant polynomial

(b) polynomial of degree $n-1$

(c) polynomial of degree n

(d) polynomial of degree $n+1$

4. What is the value of the limit

$$\lim_{x\to 0}\left[\frac{1}{1\sin^2 x}+\frac{1}{2\sin^2 x}+\ldots+\frac{1}{n\sin^2 x}\right]^{\sin^2 x}?$$

(a) ∞

(b) 0

(c) $\frac{n(n+1)}{2}$

(d) n

5. If $\sin(y+z-x)$, $\sin(z+x-y)$ and $\sin(x+y-z)$ are in AP, then $\tan x$, $\tan y$ and $\tan z$ are in

(a) HP

(b) GP

(c) AP

(d) None of the above

6. For the series 14, 28, 20, 40, 32, 64, ..., what number should come next?

(a) 56

(b) 96

(c) 52

(d) 128

7. An application was received by inward clerk in the afternoon of a weekday. Next day he forwarded it to the table of the senior clerk, who was on leave that day. The senior clerk next day evening put up the application to the desk officer. The desk officer studied the application and disposed off the matter on the same day, i.e., Friday. Which day was the application received by the inward clerk?

(a) Earlier week's Saturday

(b) Monday

(c) Tuesday

(d) Wednesday

8. The residue of $f(z) = \cot z$, at all the singular points of $f(z)$ is

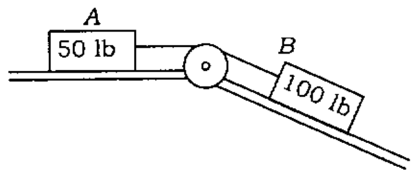
(a) -2

(b) -1

(c) 0

(d) 1

9. Two blocks A and B weighing 50 lb and 100 lb respectively are connected by an inextensible (cannot stretch) cord and are free to move on frictionless surface. The pulley is frictionless and massless. What is the measure of tension in the cord when the system is released?



(a) 20 lb

(b) 75 lb

(c) 150 lb

(d) None of the above

10. Let $f_1 : \mathbb{R} \to \mathbb{R}$, $f_2 : \mathbb{R}^2 \to \mathbb{R}$ and $f_3 : \mathbb{R}^3 \to \mathbb{R}$ be functions defined as below, where $\mathbb{R}$ denotes the set of real numbers

$$f_1(x) = \max\{0, x\},\ f_2(x_1, x_2) = 2x_1^2 + 2x_1x_2 - 8x_2 - \log(x_1x_2) \text{ and}$$
$$f_3(x_1, x_2, x_3) = \exp(x_1^2 + x_2^2 + x_3^2)$$

What is the number of functions which are convex?

(a) 0

(b) 1

(c) 2

(d) 3

11. Let S_n be the symmetric group of permutations of indices 1, 2, ..., n and let

$$\sigma : S_n \to \{-1, +1\}$$

be the sign homomorphism. Which one of the following is true?

(a) Order of S_n is n and order of $\sigma(S_n)$ is also n

(b) Order of S_n is $n!$ and order of $\sigma(S_n)$ is 2

(c) Order of S_n is n and order of $\sigma(S_n)$ is 2

(d) Order of S_n is $n!$ and order of $\sigma(S_n)$ is n

12. What is the step size that can be used in tabulation of $f(x) = \sin x$ in $\left[0, \frac{\pi}{4}\right]$ at equally spaced nodal points so that the truncation error of quadratic interpolation is less than 5×10^{-8}?

(a) $h \approx 0{\cdot}05$

(b) $h \approx 0{\cdot}1$

(c) $h \approx 0{\cdot}009$

(d) $h \approx 0{\cdot}0009$

13. Let $ABCD$ be a parallelogram. Let L and M be the middle points of AB and BC respectively. Then $DL + DM =$

(a) $\frac{3}{2} AC$

(b) $\frac{3}{2} DB$

(c) $\frac{1}{2} BD$

(d) $\frac{1}{2} CA$

14. Let n be a positive integer. Each subset of the set $\{1, 2, \ldots, 2n\}$ of size $n+1$ contains coprime numbers. Which one of the following statements is correct?

(a) There must be at least one pair of adjacent numbers

(b) There must be at least one pair of odd numbers

(c) Both of the above

(d) Insufficient information

15. The diameter of a 2-inch steel shaft is measured to the nearest thousandth of an inch and a railroad track is measured to the nearest foot. In this context, which one of the following statements is correct?

(a) The measurement of the steel shaft is less accurate than the railway track

(b) The relative error of the measurement of the shaft is less than that of the railway track

(c) The absolute errors in the two measurements are 0·005 inch and 6 inches respectively

(d) None of the above

16. What is the solution of the equation $xy' = y(\log y - \log x + 1)$?

(a) $\log\left(\frac{y}{x}\right) = cy$

(b) $\log\left(\frac{x}{y}\right) = cy$

(c) $\log\left(\frac{y}{x}\right) = cx$

(d) $\log\left(\frac{x}{y}\right) = cx$

17. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then what is the value of $\frac{\sum_{k=1}^{2}(x^{100k} + y^{106k})}{\sum x^{207} y^{207}}$?

(a) 4

(b) $\frac{1}{3}$

(c) $\frac{2}{3}$

(d) $\frac{4}{3}$

18. The solution of the system of equations $u' = Au$, where $u = [u_1, u_2]^T$ and $A = \begin{bmatrix} -3 & 4 \\ -2 & 3 \end{bmatrix}$ is given by

(a) $u_1(x) = c_1 e^x + 2c_2 e^{-x}$
$u_2(x) = c_1 e^x + c_2 e^{-x}$

(b) $u_1(x) = 2c_1 e^x + c_2 e^{-x}$
$u_2(x) = c_1 e^x + c_2 e^{-x}$

(c) $u_1(x) = c_1 e^x + 2c_2 e^{-x}$
$u_2(x) = 2c_1 e^x + c_2 e^{-x}$

(d) $u_1(x) = 2c_1 e^x + c_2 e^{-x}$
$u_2(x) = c_1 e^x + 2c_2 e^{-x}$

19. An investor has twenty thousand dollars to invest among four possible investments. Each investment must be in units of thousand dollars. If the total twenty thousand is to be invested, how many different investment strategies are possible?

(a) 10626

(b) 1651

(c) 1771

(d) 80

20. Arrange the words given below in most meaningful sequence :

1. Poverty
2. Population
3. Death
4. Unemployment
5. Disease

(a) 2, 3, 4, 5, 1

(b) 3, 4, 2, 5, 1

(c) 2, 4, 1, 5, 3

(d) 1, 2, 3, 4, 5

21. An electronic device when fed with the numbers, rearranges them in a particular order following certain rules. The following is a step-by-step process of rearrangement for the given numbers. Step *V* is the last step for any given input :

Input	:	85	16	36	04	19	97	63	09
Step I	:	97	85	16	36	04	19	63	09
Step II	:	97	85	63	16	36	04	19	09
Step III	:	97	85	63	36	16	04	19	09
Step IV	:	97	85	63	36	19	16	04	09
Step V	:	97	85	63	36	19	16	09	04

For the input 25 08 35 11 88 67 23, which one of the following is the Step *V*?

(a) 08 11 23 25 35 67 88

(b) 88 67 35 25 23 11 08

(c) 88 67 35 25 08 11 23

(d) 88 67 35 25 23 08 11

22. What is the number of points of intersection of the following two curves?

$$y = 2\sin x \text{ and } y = 5x^2 + 2x + 3$$

(a) 3

(b) 2

(c) 1

(d) 0

23. Let X is a standard normal variate. What will be the value of $E[e^X + X + 1]$?

(a) $e^{\frac{1}{2}}$

(b) $1 + e^{\frac{1}{2}}$

(c) e

(d) None of the above

24. Let the unit vectors $\bar{a}$ and $\bar{b}$ be inclined at an angle 2θ, $(0 \le \theta \le \pi)$ and that $|a - b| < 1$. In which one of the following intervals does θ lie?

(a) $\left[0, \frac{\pi}{6}\right)$

(b) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right)$

(c) $\left[\frac{\pi}{3}, \frac{\pi}{2}\right)$

(d) $\left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$

25. P, Q, R and S are the points on the line joining the points $P(a, x)$ and $T(b, y)$ such that $PQ = QR = RS = ST$. For which one of the following, $\left(\frac{5a + 3b}{8}, \frac{5x + 3y}{8}\right)$ is the midpoint?

(a) PS

(b) QR

(c) QT

(d) RT

26. The positive non-integral solution of the equation $\sin\pi(x^2 + x) = \sin\pi x^2$ is

(a) rational

(b) irrational of the form $\sqrt{p}$

(c) irrational of the form $\frac{\sqrt{p} - 1}{4}$, where p is an odd integer

(d) irrational of the form $\frac{\sqrt{p} + 1}{4}$, where p is an even integer

27. Examine the following statements :

Statement I : Trapezoidal rule is exact only for polynomials of degree less than or equal to one.

Statement II : Simpson's rule is exact only for polynomials of degree less than or equal to two.

Statement III : The Newton-Cotes formula which approximates $\int_a^b f(x)\,dx$ to be the area of the region with width $(b - a)$ and ordinates $f(a)$, $f(b)$ is Simpson's rule.

How many of the above statements are correct?

(a) One

(b) Two

(c) Three

(d) None of the above

28. The locus of the foot of the perpendicular from the origin on the variable plane through the fixed point (2, –4, 6) is a sphere. What is the radius of this sphere?

(a) $2\sqrt{14}$

(b) $\sqrt{56}$

(c) $\sqrt{14}$

(d) $2\sqrt{56}$

29. Find the odd one out in the following :

(a) Snore

(b) Dream

(c) Slumber

(d) Yawn

30. The system of equations $Ax = b$ be solved iteratively by $x_{n+1} = Mx_n + b$. Suppose, $A = \begin{bmatrix} 1 & k \\ 2k & 1 \end{bmatrix}$, $k \neq \frac{\sqrt{2}}{2}$, k is real. Then which one of the following is a necessary and sufficient condition on k for the convergence of Jacobi method?

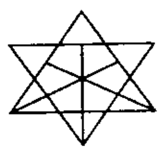
(a) $0 < k < 1$

(b) $-\frac{1}{\sqrt{2}} < k < \frac{1}{\sqrt{2}}$

(c) $-\sqrt{2} < k < \sqrt{2}$

(d) No such condition on k exists

31. How many triangles are there in the given figure?



(a) 21

(b) 23

(c) 25

(d) 27

32. Let $f(x)$ be a polynomial of degree three satisfying $f(0) = -1$ and $f(1) = 0$. Also 0 is a stationary point of $f(x)$. If $f(x)$ does not have an extremum at $x = 0$, then

$$\int \frac{f(x)}{x^3 - 1} dx =$$

(a) $\frac{x^3}{6} + C$

(b) $\frac{x^2}{2} + C$

(c) $\frac{x^2}{2} + x + C$

(d) $x + C$

33. If $k^3 - k - 1 = 0$, then what is the exact value of $(3k^2 - 4k)^{\frac{1}{3}} + k(2k^2 + 3k + 2)^{\frac{1}{4}}$?

(a) 2

(b) 1

(c) 0

(d) -4

34. If A, B, C are angles of a triangle and if $x = \tan\left(\frac{B-C}{2}\right)\tan\left(\frac{A}{2}\right)$, $y = \tan\left(\frac{C-A}{2}\right)\tan\left(\frac{B}{2}\right)$ and $z = \tan\left(\frac{A-B}{2}\right)\tan\left(\frac{C}{2}\right)$, then what is the value of $x + y + z + xyz$?

(a) -1

(b) 0

(c) 1

(d) 3

35. What is the equation of the curve passing through the point (1, 1) and whose slope is $\frac{2ay}{x(y-a)}$?

(a) $y^a x^{2a} = e^y$

(b) $y^{2a} x^a = e^y$

(c) $y^{2a} x^a = e^{y-1}$

(d) $y^a x^{2a} = e^{y-1}$

36. Let

$$[a_{ij}] = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}^n$$

then for $n = 17$ the entries a_{11}, a_{23} and a_{33} respectively are

(a) 1, 0, 17

(b) 1, 1, 16

(c) 1, 17, 153

(d) 1, 17, 136

37. The last three digits of a telephone number have been erased and all that we know is that the number is 54426???. Assuming that all possibilities are equally likely, what is the probability of the event E = {the missing digits are 7, 4, 0} (in this given order)?

(a) 0·006

(b) 0·001

(c) 0·01

(d) 0·07

38. Choose the odd numeral pair of the following :

(a) (48, 34)

(b) (55, 42)

(c) (69, 56)

(d) (95, 82)

39. What will be the order of the differential equation of family of circles touching a fixed straight line passing through origin?

(a) One

(b) Two

(c) Three

(d) None of the above

40. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5?

(a) $\frac{9}{20}$

(b) $\frac{8}{15}$

(c) $\frac{2}{5}$

(d) $\frac{1}{2}$

41. What is the arithmetic mean of the non-zero solution of the equation

$$\tan^{-1}\frac{1}{2x+1}+\tan^{-1}\frac{1}{4x+1}=\tan^{-1}\frac{2}{x^2}\ ?$$

(a) $\frac{7}{6}$

(b) $\frac{7}{3}$

(c) $\frac{2}{3}$

(d) None of the above

42. Identify the false statement from the following :

(a) Cyclic groups of prime order are compound groups

(b) An Abelian group is a group whose law of composition is commutative

(c) A finite field is a field that contains finitely many elements

(d) Rings are algebraic structures closed under addition, subtraction and multiplication

43. Two identical components operating in parallel share a common load. If one component fails, the probability that the other component fails increases as a result of the increased load placed on it. Let A and B be the events that first component fails and the event that second component fails, respectively. If $P(A)=P(B)=0{\cdot}05$ and $P(A|B)=P(B|A)=0{\cdot}10$, what is the probability that at least one of the two components fails and at least one of the two components does not fail?

(a) 0·005 and 0·905

(b) 0·008 and 0·095

(c) 0·095 and 0·995

(d) 0·009 and 0·095

44. What is the area bounded by the curve $y=\sin^{-1}x$ and the lines $x=0$, $|y|=\frac{\pi}{2}$?

(a) 1

(b) 2

(c) $\sqrt{3}$

(d) $\frac{\sqrt{3}}{2}$

45. Five friends A, B, C, D and E each bought a mobile phone for some price, the price being different for each. A paid more than both C and E. Only B paid more than D. E paid ₹ 8,000 for the same being not the minimum amount. Which of the following is true in this context?

(a) Only two persons paid the price less than that paid by B

(b) E paid more than B and C

(c) C paid the highest amount

(d) None of the above

46. Suppose p, q, r are positive integers such that

$$pqr + pq + qr + rp + p + q + r = 1000$$

What is the value of $p + q + r$?

(a) 28

(b) 7

(c) 21

(d) 14

47. The position vectors of three points A, B and C are $i + j$, $i - j$ and $li + mj + nk$ respectively. The vectors OA, OB and OC are coplanar if

(a) $l = m = n = 1$

(b) $l = 0$, $m = 1$ and n is any scalar

(c) $n = 0$, l and m are scalars

(d) $m = 1$, l and n are scalars

48. An urn contains N white and M black balls. The balls are randomly selected, one at a time, until a black one is obtained. If we assume that each selected ball is replaced before the next one is drawn, what is the probability that exactly n draws are needed?

(a) $\frac{MN^{n-1}}{(M+N)^n}$

(b) $\frac{MN^{n-1}}{(M+N)}$

(c) $\frac{MN}{(M+N)^n}$

(d) $\frac{MN^n}{(M+N)^{n-1}}$

49. Which one of the following, is different from the rest?

(a) [E]

(b) [A]

(c) [F]

(d) [Z]

50. Consider the statement

$x(\alpha - x) < y(\alpha - y)$ for all x, y with $0 < x < y < 1$

The statement is true

(a) if and only if $\alpha < -1$

(b) if and only if $\alpha < 2$

(c) if and only if $\alpha > 2$

(d) For no value of α

51. A random variate X has a Pareto distribution with parameter 4 if

$$f_X(t) = \begin{cases} kt^{-4} & t > 1 \\ 0 & t \le 1 \end{cases}$$

where k is some positive constant. What is the value of k?

(a) 1

(b) 2

(c) 3

(d) 4

52. There are seven books one each on Psychology, Hindi, English, Sociology, Economics, Education and Accountancy lying on a table one above the other. Sociology is on the top of all the books. Accountancy is immediately below Education which is immediately below Sociology. Economics is immediately above Psychology but not in the middle. Hindi is immediately below Psychology. Between which of the following books is the Economics book placed?

(a) Education and Psychology

(b) English and Psychology

(c) Accountancy and Psychology

(d) English and Hindi

53. The equation of the smallest circle passing through the intersection of the line $x + y = 1$ and the circle $x^2 + y^2 = 9$ is

(a) $x^2 + y^2 - x - y - 10 = 0$

(b) $x^2 + y^2 - x + y + 10 = 0$

(c) $x^2 + y^2 - x - y - 8 = 0$

(d) $x^2 + y^2 + x + y - 8 = 0$

54. Which one of the following recurrence relations do the Legendre polynomials $P_n(x)$ with $P_0(x) = 1$ and $P_1(x) = x$ satisfy?

(a) $(n+1)P_{n+1}(x) = (2n-1)xP_n(x) - nP_{n-1}(x)$

(b) $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$

(c) $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) + nP_{n-1}(x)$

(d) $(n+1)P_{n+1}(x) = (2n-1)xP_n(x) + nP_{n-1}(x)$

55. How many words can be formed using all the letters of the word ENTRANCE keeping the vowels together?

(a) 10080

(b) 3660

(c) 1220

(d) 1080

56. Let $I = [0, 1]$ be the closed interval and let $f : I \to \mathbb{R}$ be a real-valued function defined by

$$f(x) = \begin{cases} x & \text{when } x \text{ is rational} \\ 1 - x & \text{when } x \text{ is irrational} \end{cases}$$

The composite function $(f \circ f)(x)$ will be equal to

(a) $1 - x$ for all $x \in I$

(b) x for all $x \in I$

(c) 0 for all $x \in I$

(d) None of the above

57. In a trapezoid, *ABCD* the vector $\overline{BC} = \lambda \overline{AD}$ and $\overline{p} = \mu \overline{AD}$. Also given that $\overline{p} = \overline{AC} + \overline{BD}$ is collinear with $\overline{AD}$. Which one of the following is correct?

(a) $\lambda = \mu + 1$

(b) $\mu = \lambda + 1$

(c) $\mu + \lambda = 1$

(d) $\mu = \lambda + 2$

58. $\int_0^{\sin^2 x} \sin^{-1}(\sqrt{t})\, dt + \int_0^{\cos^2 x} \cos^{-1}(\sqrt{t})\, dt =$

(a) π

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$

(d) 0

59. The expected value of number of times one must throw a die until the outcome 1 has occurred four times is

(a) 40

(b) 24

(c) 12

(d) 8

60. Let a rectangular region with vertices $O(0, 0)$, $A(1, 0)$, $B(1, 2)$, $C(0, 2)$ be defined in the z-plane. Which one of the following mappings is true for the image of the region in the w-plane under the mapping $w = (1 - i)z - 2i$?

(a) Only translation

(b) Composition of translation and magnification

(c) Composition of translation and rotation

(d) Composition of translation, magnification and rotation

61. Which one of the following triplets cannot represent direction cosines of a line?

(a) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

(b) $\left(\frac{5}{\sqrt{50}}, \frac{3}{\sqrt{50}}, \frac{4}{\sqrt{50}}\right)$

(c) $\left(\frac{7}{\sqrt{66}}, \frac{4}{\sqrt{66}}, \frac{1}{\sqrt{66}}\right)$

(d) $\left(\frac{5}{\sqrt{75}}, \frac{6}{\sqrt{75}}, \frac{4}{\sqrt{75}}\right)$

62. A sprinter competing in a 100-metre race accelerates uniformly for the first 40 metres in 6·1 seconds. She then runs at a constant speed for the remainder of the race. At what time does she cross the finishing line?

(a) 10·675 seconds

(b) 10·721 seconds

(c) 09·991 seconds

(d) 10·052 seconds

63. A 15 kg piece of wood is placed on top of another piece of wood. There is 35 N of static friction measured between them. What is the approximate value of the coefficient of static friction between the two pieces of wood?

(a) 4·20 N

(b) 0·24 N

(c) 0·10 N

(d) None of the above

64. A rigid motion of a plane carrying a subset F of the plane itself is called

(a) the symmetry of F

(b) the isometry of F

(c) the rotation of F

(d) the translation of F

65. A smoke detector is routinely inspected. 80% of the detectors found inoperative had experienced a power surge and 10% of those found in operating condition had experienced a power surge. 20% of the detectors inspected have failed. What is the probability of a detector failing given it experiences a power surge?

(a) 0·80

(b) 0·10

(c) 0·167

(d) 0·667

66. Let the set of real numbers be denoted by $\mathbb{R}$, the n-space by $\mathbb{R}^n$ and the set of complex numbers be denoted by $\mathbb{C}$. Which one of the following is the incorrect statement?

(a) $\mathbb{R} \subset \mathbb{C}$

(b) For any vectors $u, v \in \mathbb{R}^n$, the Cauchy-Schwarz inequality is $|u.v| \leq ||u|| \, ||v||$

(c) For a complex number $z = a + bi$, a is the real part and b is the imaginary part

(d) The complex number denoted by a point $(-1, 0)$ in the plane is denoted by i, where $i = \sqrt{-1}$

67. If

$$D_r = \begin{vmatrix} r & x & \frac{n(n+1)}{2} \\ 2r-1 & y & n^2 \\ 3r-2 & z & \frac{n(3n-1)}{2} \end{vmatrix}$$

then what is the value of $\sum_{r=1}^{n} D_r$?

(a) n

(b) $n(n+1)$

(c) n^2

(d) 0

68. If $x_1 = 2$ and $x_{n+1} = 2 + \frac{1}{x_n}$ for $n \geq 1$, then what is the limit of the sequence $\{x_{n+1}\}$?

(a) 2

(b) $1+\sqrt{2}$

(c) $1+2\sqrt{2}$

(d) ∞

69. The non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$ and $\vec{c} = -7\vec{b}$. What is the angle between $\vec{a}$ and $\vec{c}$?

(a) π

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$

(d) 0

70. For which value of θ, the expression $2^{\sin\theta} + 2^{-\cos\theta}$ is minimum?

(a) $\theta = 2n\pi + \frac{\pi}{4}, n \in I$

(b) $\theta = 2n\pi + \frac{3\pi}{4}, n \in I$

(c) $\theta = 2n\pi + \frac{3\pi}{2}, n \in I$

(d) $\theta = 2n\pi + \frac{7\pi}{4}, n \in I$

71. How many times do the hands of a clock coincide in a day?

(a) 20

(b) 21

(c) 22

(d) 24

72. The rate of convergence of Gauss-Seidel method is

(a) same as Jacobi method

(b) 1·5 times the Jacobi method

(c) 2 times the Jacobi method

(d) 3 times the Jacobi method

73. Let two tangents are drawn from the point (x_1, y_1) to the parabola $y^2 = 4x$. Which one of the following is true if the slope of one tangent be double of that of the other?

(a) $4x_1^2 = 9y_1$

(b) $2x_1^2 = 9y_1$

(c) $4y_1^2 = 9x_1$

(d) $2y_1^2 = 9x_1$

74. If $E(\theta) = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$, and θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$, then $E(\theta)E(\phi)$ is a

(a) diagonal matrix

(b) null matrix

(c) unit matrix

(d) None of the above

75. The sequence $\{(-1)^n\}$ is a

(a) convergent sequence

(b) Cauchy sequence

(c) bounded sequence

(d) All of the above

76. A square *ABCD* of diagonal $2a$ is folded along the diagonal *AC* so that the planes *DAC* and *BAC* are at right angle. What is the shortest distance between *DC* and *AB*?

(a) $(\sqrt{3}/2)a$

(b) $(\sqrt{2})a$

(c) $2a/\sqrt{3}$

(d) $2a/\sqrt{5}$

77. Let $A * B$ mean A is greater than B; $A \underline{*} B$ mean A is either greater than or equal to B; $A = B$ mean A is equal to B; $A \# B$ mean A is smaller than B, and $A \underline{\#} B$ mean A is either smaller than or equal to B.

Given $M = N$, $N * B$ and $B \# P$.

Inferences :

I. $P = N$

II. $B \# M$

Which one of the following is true?

(a) Only inference I is true

(b) Only inference II is true

(c) Both inferences I and II are true

(d) Neither inference I nor inference II is true

78. If x, y, z and w satisfy the equations

$$x + 7y + 3z + 5w = 0$$
$$8x + 4y + 6z + 2w = -16$$
$$2x + 6y + 4z + 8w = 16$$
$$5x + 3y + 7z + w = -16$$

then $(x + w)(y + z) =$

(a) 16

(b) 3

(c) 0

(d) None of the above

79. Which one of the following is the smallest interval in which the eigenvalues of the matrix

$$\begin{bmatrix} 3 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

lie?

(a) $2 \leq \lambda \leq 4$

(b) $-1 \leq \lambda \leq 6$

(c) $0 \leq \lambda \leq 7$

(d) $1 \leq \lambda \leq 8$

80. If $x > 0$ and g is a bounded function, then $\lim_{n \to \infty} \dfrac{f(x)e^{nx} + g(x)}{e^{nx} + 1}$ is

(a) 0

(b) $g(x)$

(c) $f(x)$

(d) None of the above

81. If LBAEHC is the code for BLEACH, then which one of the following is coded as NBOLZKMH?

(a) BNLOKZHM

(b) MANKYJLG

(c) LOBNHMKZ

(d) OBNKZLHM

82. A mirror and a source of light are situated at the origin O and at a point on OX respectively. A ray of light from the source strikes the mirror and is reflected. If the direction ratios of the normal to the plane are 1, −1, 1, then what are the direction cosines of the reflected rays?

(a) $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

(b) $-\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}$

(c) $\frac{2}{3}, -\frac{2}{3}, \frac{2}{3}$

(d) $\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}$

83. Consider the following two statements :

Statement I :

Consider the function $g:[-1,\ 1]\rightarrow\mathbb{R}$ defined by

$$g(x)=\begin{cases}1 & \text{for} \quad 0<x\leq 1\\ 0 & \text{for} \quad x=0\\ -1 & \text{for} \quad -1\leq x<0\end{cases}$$

Then there exists a function f such that $f'(x)=g(x)$ for all x in $[-1,\ 1]$, where f' is the derivative of f.

Statement II :

Let I be an interval and let $h:I\rightarrow\mathbb{R}$ be differentiable on I. If the derivative h' is never 0 on I, then either $h'(x)>0$ for all x in I or $h'(x)<0$ for all x in I.

Identify the correct option from the following :

(a) Both statements I and II are true

(b) Only statement I is true

(c) Only statement II is true

(d) Both statements I and II are false

84. What is the acceleration of a 20·0 kg curling stone if the applied force is 88·7 N West and the force of friction is 29·73 N?

(a) 2·87 m/s^2 East

(b) 2·95 m/s^2 East

(c) 5·92 m/s^2 West

(d) 2·95 m/s^2 West

85. If $e^{-\pi/2} < \theta < \frac{\pi}{2}$, then which one of the following relations is true?

(a) $\cos\log\theta < \log\cos\theta$

(b) $\cos\log\theta > \log\cos\theta$

(c) $\cos\log\theta \le \log\cos\theta$

(d) None of the above

86. Consider the parabola $y^2 = 4x$; $A = (4, -4)$ and $B = (9, 6)$ be two fixed points on the parabola. Let C be a moving point on the parabola between A and B. What are the coordinates of C such that the area of the triangle ABC is maximum?

(a) $\left(\frac{1}{4}, 1\right)$

(b) $(4, 4)$

(c) $(3, 2\sqrt{3})$

(d) $(3, -2\sqrt{3})$

87. For two vectors $\vec{a}$ and $\vec{b}$ if $\vec{a}.\vec{b} < 0$ and $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$, then what is the angle between vectors $\vec{a}$ and $\vec{b}$?

(a) $\frac{7\pi}{4}$

(b) $\frac{3\pi}{4}$

(c) $\frac{\pi}{4}$

(d) π

88. The equation $x^3 - 7x^2 + 16x - 12 = 0$ has a double root at $x = 2$. Starting with $x_0 = 1$, Newton-Raphson method is applied to find a root of the above equation. The order of convergence of Newton-Raphson method for the above problem is

(a) 1

(b) 2

(c) 1·5

(d) 0·75

89. A man invests ₹ 10,000 a year. Of this ₹ 4,000 is invested at the interest rate of 5% per year, ₹ 3,500 at 4% per year and the rest at α% per year. His total interest for the year is ₹ 500. What is the value of α?

(a) 6·4

(b) 6·35

(c) 6·3

(d) 6·25

90. What is the shortest distance between the line $y - x = 1$ and the curve $x = y^2$?

(a) $\frac{4}{\sqrt{3}}$

(b) $\frac{\sqrt{3}}{4}$

(c) $\frac{4}{3\sqrt{2}}$

(d) $\frac{3\sqrt{2}}{8}$

91. Let A and B be acute positive angles satisfying

$$3\sin^2 A + 2\sin^2 B = 1 \text{ and } 3\sin 2A - 2\sin 2B = 0$$

Then which one of the following is true?

(a) $A = \frac{\pi}{4} - 2B$

(b) $A = \frac{\pi}{4} - \frac{B}{2}$

(c) $B = \frac{\pi}{4} - \frac{A}{2}$

(d) $B = \frac{\pi}{2} - \frac{A}{4}$

92. Let $\{f_n\}$ be the Fibonacci sequence of numbers defined by

$$f_1 = 1, f_2 = 1 \text{ and } f_n = f_{n-1} + f_{n-2} (n \geq 2)$$

Let $x_n = \frac{f_{n+1}}{f_n}$. The sequence $\{x_n\}$ converges to

(a) ∞

(b) 1

(c) $\sqrt{2} + 1$

(d) None of the above

93. On receipt of a shipment of 5 boards, select 2 boards for inspection at random. If boards 1 and 2 are the only defective boards and X is the number of defective boards selected, then what is the probability mass function for X when $X = 1$?

(a) 0·1

(b) 0·2

(c) 0·3

(d) 0·6

94. Let λ and α be real. Consider the system of linear equations

$$\lambda x + (\sin\alpha)y + (\cos\alpha)z = 0$$
$$x + (\cos\alpha)y + (\sin\alpha)z = 0$$
$$-x + (\sin\alpha)y - (\cos\alpha)z = 0$$

The set of all values of λ for which the above system of linear equations has a non-trivial solution is

(a) $[-1, 1]$

(b) $[-\sqrt{2}, 0]$

(c) $[0, \sqrt{2}]$

(d) $[-\sqrt{2}, \sqrt{2}]$

95. The two conics $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ and $y^2 = -\frac{b}{a}x$ intersect if

(a) $0 < a \leq \frac{1}{2}$

(b) $0 < b \leq \frac{1}{2}$

(c) $b^2 < a^2$

(d) $b^2 > a^2$

96. If a, b, c form an AP with common difference $d(\neq 0)$ and x, y, z form a GP with common ratio $r(\neq 0)$, then the area of the triangle with vertices (a, x), (b, y) and (c, z) is independent of

(a) r

(b) x

(c) d

(d) b

97. The unit normal vector to the surface $xy^2 + 2yz = 8$ at the point (3, –2, 1) is

(a) $\dfrac{3i - 2j + k}{\sqrt{14}}$

(b) $\dfrac{4i - 10j - 4k}{\sqrt{33}}$

(c) $\dfrac{2i - 5j - 2k}{\sqrt{33}}$

(d) $\dfrac{6i - 4j + k}{\sqrt{53}}$

98. Let any vectors $u, v, w \in \mathbb{R}^3$ be $u = (1, 3, -1)$, $v = (2, 0, 1)$ and $w = (1, -1, 1)$. Which one of the following is true?

(a) Vectors are dependent

(b) Vectors are independent

(c) The system has only the zero

(d) The system is inconsistent

99. Identify the incorrect one of the following statements :

(a) 2 is a significant figure in the number 0·00263

(b) 2·7183 is an approximation to the true value of e

(c) $\dfrac{1}{3}$ is an exact number

(d) The value of π when rounded off to four figures is 3·1416

100. Every integer of the form $(n^3 - n)(n-2)$ for $n = 3, 4, \ldots$ is

(a) divisible by 6 but not always divisible by 12

(b) divisible by 12 but not always divisible by 24

(c) divisible by 24 but not always divisible by 48

(d) divisible by 9

101. In a class 60% of students passed in English, 50% of students passed in Hindi. Then every student passed in either English or Hindi or both the subjects and 120 numbers of students passed in both the subjects. How many students, in number, are there in the class?

(a) 1000

(b) 1200

(c) 950

(d) 1500

102. Given a vector field $\mathbf{F} = 2x(y^2 + x^3)\mathbf{i} + 2x^2 y\mathbf{j} + 3x^2 z^2\mathbf{k}$. What is the scalar potential of the vector field **F**?

(a) $x^2(y^2 + z^3) + c$

(b) $z(y^2 + x^3) + c$

(c) $y^3(x^2 + z^3) + c$

(d) None of the above

103. What is the area of the quadrilateral formed by the tangents at the end of the latus rectum to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$?

(a) 27 sq. units

(b) 27/2 sq. units

(c) 9 sq. units

(d) 27/4 sq. units

104. Two children are playing on a seesaw of length 3·5 metres. Each of the children weighs 30 kg and 35 kg respectively. Their centre of mass is at

(a) 1·75

(b) 1·3

(c) 1·8

(d) None of the above

105. If $I = \int (\log_e x)^n \, dx$, then what is the value of $I_n + nI_{n-1}$?

(a) $(\log_e x)^n$

(b) $(\log_e x)^{n-2}$

(c) $x(\log_e x)^n$

(d) $x(\log_e x)^{n-1}$

106. How many pairs of letters are there in the word SEQUENTIAL which have as many letters between them in the word as in the alphabet?

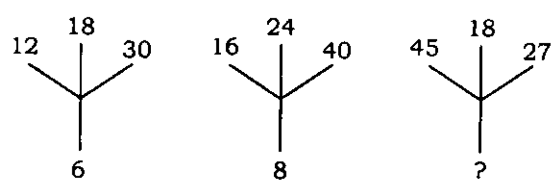
(a) Four

(b) Three

(c) Two

(d) None of the above

107. Which one of the given numbers should replace the question mark in the following figure?



(a) 6

(b) 9

(c) 12

(d) 18

108. Given five numbers 98, 99, 100, 101, 102. What is the variance?

(a) 4

(b) 3

(c) 100

(d) None of the above

109. $\lim_{t \to \infty} \sum_{r=1}^{t} \cot^{-1}\left(r^2 + \frac{3}{4}\right) =$

(a) 0

(b) $\tan^{-1} 1$

(c) $\tan^{-1} 2$

(d) None of the above

110. If α and β are two distinct roots of the equation $a\cos x + b\sin x = c$, then what is the value of $\tan\left(\frac{\alpha+\beta}{2}\right)$?

(a) $\frac{a}{b}$

(b) $\frac{b}{a}$

(c) $\frac{a}{c}$

(d) $\frac{c}{a}$

111. If

$$f(x) = \begin{cases} e^x & , \quad x < 0 \\ a + bx & , \quad x \geq 2 \end{cases}$$

is differentiable for all $x \in \mathbb{R}$, then which one of the following is true?

(a) $2a + b = 0$

(b) $2b = e^2$

(c) $a + 2b = e^2$

(d) None of the above

112. The value of $(256)^{0 \cdot 16}(16)^{0 \cdot 18}$ is

(a) 4

(b) 16

(c) 64

(d) 256·25

113. Let $f(x) = (x^2 - 1)^{n+1}(x^2 + x + 1)$. For which value of n, $f(x)$ has local extremum at $x = 1$?

(a) $n = 2$

(b) $n = 6$

(c) $n = 4$

(d) $n = 5$

114. Consider any vector $u \in \mathbb{C}^5$. For which one of the following options $u.u \neq 0$?

(a) $u = (1, i, 1, i, 1)$

(b) $u = (i, 1, i, 1, 0)$

(c) $u = (1, i, 0, 1, i)$

(d) $u = (0, 0, i, 1, 0)$

115. In the series

6 4 1 2 2 8 7 4 2 1 5 3 8 6 2 1 7 1 4 1 3 2 8 6

how many pairs of alternate numbers have the difference of 2?

(a) Four

(b) Three

(c) Two

(d) One

116. The synonym of Adept is

(a) Accomplished

(b) Awkward

(c) Indefinite

(d) Inept

117. Consider the random polynomial $p(x) = x^2 + Bx + C$. What is the probability that p has real roots?

(a) $\frac{1}{3}$

(b) $\frac{1}{4}$

(c) $\frac{1}{8}$

(d) $\frac{1}{12}$

118. If m men can do a job in D days, then in how many days in which $(m+r)$ men can do the job?

(a) $D+r$

(b) $\frac{D}{m+r}$

(c) $\frac{mD}{m+r}$

(d) $\frac{D}{m}(m+r)$

119. If $f(x) = \max(\{1-\cos x, 2\sin x\})$, $\forall x \in (0, \pi)$, then $f'(x)$ is not defined at

(a) $x = \frac{5\pi}{12}$

(b) $x = x_0$, where $2\sin x_0 + \cos x_0 = 1$

(c) $x = \frac{\pi}{12}$

(d) $x = \frac{\pi}{2}$

120. Between which of the following two values does the equation $x^3 - 7x - 1 = 0$ have a solution?

(a) $-0{\cdot}3$ and $-0{\cdot}2$

(b) $-0{\cdot}2$ and $-0{\cdot}1$

(c) $-0{\cdot}1$ and 0

(d) 0 and $0{\cdot}1$

★★★